Lesson 7

Powers

Basic Algebra and Geometry

POWERS

Topic One: Definition of a Power

Sometimes a factor appears more than once in a product. For example, in the product 5×5, the factor 5 appears twice. This product could be written more compactly as 5^2 (which is read "5 squared"). The expression 5^2 is called a POWER. In the power 5^2, the large numeral 5 is called the BASE of the power. The smaller raised numeral 2 is the EXPONENT of the power. The exponent 2 tells us that the base 5 is to be used twice as a factor.

$$5^2 = \underbrace{5 \times 5}$$

base — exponent — The base 5 is used twice as a factor.

In general, any number which can be expressed by means of a base and an exponent is called a POWER. Any number which is written as a power is said to be in EXPONENTIAL FORM.

With respect to the power 3^4, what is the base of the power? _____

What is the exponent? _____ This power tells us that the number _____ is to be used _______ times as a factor.

i.e. $3^4 = 3 \times 3 \times 3 \times 3 =$ _____

Write $(7 \times 7 \times 7 \times 7 \times 7 \times 7)$ in exponential form. _________

EXAMPLES:

3 ← This is 3 to the first power. Note that the exponent 1 is seldom written.

$(\sqrt{2})^2$ ← This is $\sqrt{2}$ squared. It means that $\sqrt{2}$ is to be used as a factor twice.

i.e. $(\sqrt{2})^2 = \sqrt{2} \times \sqrt{2} =$ _____

$(-3)^3$ ← This is (-3) cubed. It means that (-3) is to be used as a factor 3 times.

i.e. $(-3)^3 = (-3)(-3)(-3) =$ _____

$\left(\frac{-2}{5}\right)^4$ ← This is $\frac{-2}{5}$ to the fourth power. It means that $\left(\frac{-2}{5}\right)$ is to be used as a factor 4 times.

i.e. $\left(\frac{-2}{5}\right)^4 = \frac{-2}{5} \times \frac{-2}{5} \times \frac{-2}{5} \times \frac{-2}{5} =$ _____

When writing powers, always use parentheses around negative bases. Omitting the parentheses can result in ambiguity. For example, the expressions $(-4)^2$ and -4^2 are interpreted in two different ways.

$(-4)^2$ is a power with a base of -4 and exponent of 2.	-4^2 is the additive inverse of the power 4^2.
i.e. $(-4)^2 = (-4)(-4)$ $= 16$	i.e. $-4^2 = -(4 \times 4)$ $= -16$

How would you write -2 to the fifth? _____ -7 squared? _____

What does $(-3)^4$ equal? _____ What does -3^4 equal? _____

All powers with positive bases represent positive numbers. For example,

1. $12^2 = 12 \times 12 = 144$
2. $5^4 = \_\_ \times \_\_ \times \_\_ \times \_\_ = \_\_\_\_$

The results are positive.

Powers with negative bases may be positive or negative, depending on whether the exponent is an even or odd number. If the exponent is even, the power represents a positive number. For example,

exponent is even

1. $(-2)^2 = (-2)(-2) = 4$
2. $(-3)^4 = (-3)(-3)(-3)(-3) = \_\_\_\_$
3. $(-1)^6 = (\_\_)(\_\_)(\_\_)(\_\_)(\_\_)(\_\_) = \_\_\_\_$

The results are positive.

If the exponent is odd, the power represents a negative number. For example,

exponent is odd.

1. $(-2)^3 = (-2)(-2)(-2) = -8$
2. $(-3)^5 = (-3)(-3)(-3)(-3)(-3) = \_\_\_\_$
3. $(-1)^7 = \_\_\_\_\_\_\_\_ = \_\_\_\_$

The results are negative.

Give four examples of powers with negative bases that represent positive numbers.

$(-16)^8$, _____, _____, _____

Give four examples of powers with negative bases that represent negative numbers.

$(-16)^7$, _____, _____, _____

NOTE: $(-1)^n = -1$ if n is odd. i.e. $(-1)^3 = (-1)^5 = (-1)^7 = (-1)^9 = \ldots = -1$	$(-1)^n = 1$ if n is even. i.e. $(-1)^2 = (-1)^4 = (-1)^6 = (-1)^8 = \ldots = 1$

Parentheses must also be used around fractional bases. For example, in writing $\frac{3}{4}$ cubed, we use parentheses around the base $\frac{3}{4}$ so that it is clear that the exponent 3 does not apply to the numerator only. That is, the expressions $\left(\frac{3}{4}\right)^3$ and $\frac{3^3}{4}$ are interpreted in two different ways.

$\left(\frac{3}{4}\right)^3$ is a power with a base of $\frac{3}{4}$ and exponent of 3.	$\frac{3^3}{4}$ is a fraction with a numerator of 3^3 and denominator of 4.
i.e. $\left(\frac{3}{4}\right)^3 = \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}$ $= \frac{27}{64}$	i.e. $\frac{3^3}{4} = \frac{3 \times 3 \times 3}{4}$ $= \frac{27}{4}$

How would you write $\frac{1}{2}$ squared? _____ $\frac{-3}{8}$ to the fourth? _____

What does $\left(\frac{2}{3}\right)^5$ equal? _____ What does $\frac{2^5}{3}$ equal? _____

At times you must simplify numerical expressions involving powers. Always evaluate powers first before you perform any other arithmetic operations.

EXAMPLE: Simplify the numerical expression $\frac{12 + 8^2}{-4}$

Solution

Evaluate the power 8^2 first.

$$\frac{12 + 8^2}{-4} = \frac{12 + (8 \times 8)}{-4}$$
$$= \frac{12 + 64}{-4}$$
$$= \frac{76}{-4}$$
$$= -19$$

Now, simplify the expression $(-2)^3 + 3^2$.

$$(-2)^3 + 3^2 = (-2 \times -2 \times -2) + (3 \times 3)$$
$$= \_\_\_\_\_ + \_\_\_\_\_$$
$$= \_\_\_\_\_$$

Any power that has a base which is a real number and an exponent that is a positive integer is called a POSITIVE INTEGRAL POWER. The following are all examples of positive integral powers.

3^2, $(-2)^5$, $\left(\frac{1}{2}\right)^4$, π^3, $(2.5)^4$, 9^8, $(\sqrt{2})^7$

Which of the powers above have bases that are natural numbers?

____

Which have bases that are integers? 3^2, ____, ____

Which have bases that are rational numbers? 3^2, ___, ___, ___, ___

Which have bases that are irrational numbers? ____________

EXERCISE - Positive Integral Powers

1. Fill in the blanks.

(a) With reference to the power $(2\pi)^4$, the __________ is 2π and the __________ is 4.

(b) The power $(-9)^7$ means that the base which is ______ must be used 7 times as a __________.

(c) When the number 8 is written as 2^3, it is in __________ form.

(d) When writing powers, brackets must always be used around __________ or __________ bases.

(e) If a power has a negative base and an exponent that is an ________ number, it represents a positive number.

(f) A positive integral power has a base that is a __________ number and an exponent that is a __________ integer.

2. For each of the following powers, name the base, name the exponent, and evaluate the power.

	Power	Base	Exponent	Evaluation
(a)	$\left(\frac{-3}{2}\right)^5$	$\frac{-3}{2}$	5	$\frac{-3}{2} \times \frac{-3}{2} \times \frac{-3}{2} \times \frac{-3}{2} \times \frac{-3}{2} = \frac{-243}{32}$
(b)	9^3			
(c)	$(-8)^3$			
(d)	$\left(\frac{-5}{6}\right)^2$			
(e)	$\left(\frac{-4}{7}\right)^3$			
(f)	$(\sqrt{2})^3$			
(g)	$(-\sqrt{3})^4$			
(h)	$(-1)^{18}$			$(-1)^{18} =$ ______
(i)	$(-1)^{33}$			$(-1)^{33} =$ ______

3. Decide whether each of the following expressions represents a positive or negative number.

THIS PART IS POSITIVE SINCE 12 IS EVEN.

(a) $-(-15)^{12}$ negative

(b) $(-6)^{10}$ ______

(c) $(-5)^{19}$ ______

(d) -6^2 ______

(e) $(-6)^2$ ______

(f) $(-3)^3$ ______

(g) $(-5)^{80}$ ______

(h) $(-16)^{15}$ ______

(i) $-(-1)^4$ ______

(j) $(-1)^{43}$ ______

(k) $-(-1)^7$ ______

(l) $-(-5)^2$ ______

4. Write each of the following products using exponential notation. (Use parentheses around negative or fractional bases.)

(a) $2 \times 5 \times 2 \times -7 \times -7 \times 2 \times 5 = \underline{2^3 \times 5^2 \times (-7)^2}$ } THE EXPONENTS TELL HOW MANY TIMES EACH BASE IS USED AS A FACTOR.

(b) $-6 \times -6 \times -6 \times -6 =$ ____________

(c) $8 \times 8 \times 8 \times 8 \times 5 \times 5 \times 5 \times -1 \times -1 =$ ____________

(d) $-3 \times 2 \times 3 \times 3 \times 2 \times 3 \times -3 =$ ____________

(e) $\frac{-1}{2} \times \frac{1}{3} \times \frac{-1}{2} \times \frac{1}{3} \times \frac{-1}{2} \times \frac{1}{3} \times \frac{-1}{2} =$ ____________

(f) $\frac{-4}{3} \times \frac{3}{5} \times \frac{3}{5} \times \frac{-4}{3} \times \frac{-4}{3} \times \frac{-4}{3} \times \frac{-4}{3} =$ ____________

5. Simplify each numerical expression. (Watch signs very carefully.)

(a) $7^2 - (-2)^3$

$= (7 \times 7) - (-2 \times -2 \times -2)$

$=$ ___ $-$ ___

$=$ ___ $+$ ___

$=$ ___

(b) $(-3)^2 - 2^4$

$=$

(c) $\left(\frac{-1}{2}\right)^3 - \left(\frac{3}{8}\right)^2$

$=$

(d) $-5(3^4 - 4^3)$

$=$

(e) $\frac{-5}{2} \div \left(\frac{1}{2}\right)^2$

$=$

(f) $\frac{(-1)^7 + (-2)^4}{3}$

$=$

Topic Two: Power Properties

A number of properties of powers have been developed in order to simplify work with powers. These properties are particularly useful when letters symbols (or variables) are involved. In this lesson, we will deal only with powers that have real number bases, but in Lesson 8, variable bases will be introduced.

A. Product of Powers Property

A product of powers can be written as a single power if the factors in the product have the same base.

EXAMPLE: Express the product $(2^3 \times 2^4)$ as a single power.

Solution

$$2^3 \times 2^4 = \underbrace{(2 \times 2 \times 2)}_{\text{3 factors}} \times \underbrace{(2 \times 2 \times 2 \times 2)}_{\text{4 factors}}$$

$$= \underbrace{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}_{\text{7 factors}}$$

$$= 2^7$$

Note that the same result could have been obtained by retaining the common base 2 and adding the exponents of the factors.

i.e. $2^3 \times 2^4 = 2^{3+4} = 2^7$

In general, the product of two powers with the same base can be written as a single power by retaining the common base and adding the exponents.

PRODUCT OF POWERS PROPERTY
For all positive integers m and n and any real number a, $a^m \cdot a^n = a^{m+n}$

Use the Product of Powers Property to write each product below as a single power.

1. $(-3)^7 \times (-3)^4 = (-3)^{7+4} = (-3)^{11}$

2. $6^5 \times 6^2 = 6^{\_+\_} = 6^{\_}$

3. $\left(\frac{-2}{3}\right)^2 \left(\frac{-2}{3}\right)^3 = \left(\frac{-2}{3}\right)^{\_+\_} = \left(\frac{-2}{3}\right)^{\_}$

4. $(-8)(-8)^7 = (-8)^{1+\_} = (-8)^{\_}$

Exponent of first factor is 1.

Remember that the Product of Powers Property applies ONLY when the bases of the powers are the SAME. For example, this property cannot be used to write the product $2^5 \times 3^4$ as a single power since one power has the base 2 and the other has the base 3.

Put a check mark beside each product below that can be expressed as a single power by using the Product of Powers Property.

1. $3^3 \times 3^7$ ✓ 2. $6^7 \times 7^6$ ____ 3. $(-2)^5\, 5$ ____

4. $\left(\frac{1}{2}\right)^2\left(\frac{3}{2}\right)^3$ ____ 5. $\left(\frac{-2}{3}\right)\left(\frac{-2}{3}\right)$ ____ 6. $(-3)^7\left(\frac{1}{3}\right)^5$ ____

The Product of Powers Property can be expanded to cover any number of factors with like bases. For example,

$$(\sqrt{2})^3\,(\sqrt{2})^5\,(\sqrt{2})^7\,(\sqrt{2}) = (\sqrt{2})^{3+5+7+1} = (\sqrt{2})^{16}$$

NOTE THE EXPONENT OF THE LAST FACTOR IS 1.

Similarly,

$$(-3)^6\,(-3)^4\,(-3)\,(-3)^7 = (-3)^{6+\_\_+\_\_+\_\_} = (-3)^{\_\_}$$

Self-correcting Exercise #1

Answers to this exercise may be found on page 47 of this lesson.

1. Where possible, express each of the following products as a single power.

(a) $3^7 \times 3^3$

(b) $(-2)^5 \times (-2)^{12}$

(c) $3^5 \times 5^3$

(d) $\left(\frac{1}{2}\right)^2\left(\frac{3}{5}\right)^3$

(e) $\pi^2 \times \pi$

(f) $\sqrt{3} \times (\sqrt{3})^2 \times (\sqrt{3})^3$

(g) $\left(\frac{3}{8}\right)^7 \times \left(\frac{3}{8}\right)^3 \times \left(\frac{3}{8}\right)^6$

(h) $2^3 \times 3^2$

B. Power of a Product Property

A power whose base is written as an indicated product can be written as a product of powers.

EXAMPLE: Express the power $[-2 \times 7]^3$ as the product of powers.

Solution

$$[-2 \times 7]^3 = \underbrace{(-2 \times 7)(-2 \times 7)(-2 \times 7)}_{\text{3 factors}}$$

$$= \underbrace{-2 \times -2 \times -2}_{\text{3 factors}} \times \underbrace{7 \times 7 \times 7}_{\text{3 factors}}$$

$$= (-2)^3 \times 7^3$$

Note that the same result could have been obtained by applying the exponent 3 to each factor in the product (-2×7).

i.e. $[-2 \times 7]^3 = (-2)^3 \times 7^3$

In general, a power whose base is written as an indicated product can be written as the product of powers by applying the exponent to each factor in the base.

> **POWER OF A PRODUCT PROPERTY**
>
> For the positive integer m and any real numbers a and b,
>
> $(a \times b)^m = a^m b^m$

Use the Power of a Product Property to write each expression below as a product of powers.

1. $(3 \times -2)^8 = 3^{\underline{8}} \times (-2)^{\underline{8}}$

2. $(-4 \times -7)^{10} = (\_\_)^{10} (\_\_)^{10}$

3. $(-8 \times 7)^4 = (-8)^{\_} \times \_\_^4$

4. $\left(-5 \times \frac{2}{3}\right)^3 =$

The Power of a Product Property can be expanded to cover any number of factors that make up the base of a power.
For example,

$$\left(\frac{3}{4} \times -5 \times 7 \times \frac{-2}{3}\right)^5 = \left(\frac{3}{4}\right)^5 (-5)^5 \; 7^5 \left(\frac{-2}{3}\right)^5$$

Similarly,

$$\left(-3 \times 2 \times \frac{1}{2} \times \frac{5}{7} \times -4\right)^3 = (-3)^3 \, 2^3 \underline{\hspace{10em}}$$

<u>Self-correcting Exercise #2</u>

Answers to this exercise may be found on page 47 of this lesson.

1. Express each of the following powers as a product of powers. (Brackets <u>must</u> be used around negative or fractional bases.)

(a) $(5 \times 6)^9 =$

(b) $(7 \times 8 \times 9)^{12} =$

(c) $(-2 \times \pi)^3 =$

(d) $\left(\frac{1}{2} \times \frac{3}{5} \times \frac{-7}{8}\right)^5 =$

(e) $(-\sqrt{3} \times \sqrt{5} \times \sqrt{7} \times \sqrt{10})^2 =$

(f) $(-3 \times \frac{7}{11} \times 0.2)^6 =$

C. Power of a Power Property

A power of a power can be written with a single exponent.

EXAMPLE: Express the power $(3^4)^2$ with a single exponent.

Solution

$$
\begin{aligned}
(3^4)^2 &= \underbrace{3^4 \times 3^4}_{\text{2 factors}} \\
&= \underbrace{(3 \times 3 \times 3 \times 3)}_{\text{4 factors}} \underbrace{(3 \times 3 \times 3 \times 3)}_{\text{4 factors}} \\
&= \underbrace{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}_{\text{8 factors}} \\
&= 3^8
\end{aligned}
$$

Note that the same result could have been obtained by retaining the same base 3, and multiplying the two exponents.

i.e. $(3^4)^2 = 3^{4\times 2} = 3^8$

In general, a power of a power can be written with a single exponent by retaining the same base and multiplying the exponents.

> POWER OF A POWER PROPERTY
>
> For all positive integers m and n and any real number a,
>
> $(a^m)^n = a^{mn}$

Use the Power of a Power Property to write each expression below as a single power.

1. $[(-2)^3]^4 = (-2)^{3 \times 4} = (-2)^{12}$

2. $(6^2)^{15} = (6)^{\_ \times \_} = 6^{\_}$

3. $\left[\left(\frac{1}{2}\right)^4\right]^5 = \left(\frac{1}{2}\right)^{\_\_} = \left(\frac{1}{2}\right)^{\_\_}$

4. $(7^3)^2 = \_\_^{\_ \times \_} = \_\_\_$

The Power of a Power Property can be expanded to cover any number of exponents. For example,

$$\left[(3^3)^4\right]^5 = 3^{3\times4\times5} = 3^{60}$$

Similarly,

$$\left[(2^3)^2\right]^8 = 2^{\_\_\_\_\_\_\_\_\_\_} = 2^{\_}$$

Self-correcting Exercise #3

Answers to this exercise may be found on page 47 of this lesson.

1. Express each power with a single exponent.

(a) $(0.5^2)^{14} =$

(b) $\left[(-2)^4\right]^2 =$

(c) $\left[(7^5)^6\right]^2 =$

(d) $\left[(-\sqrt{2})^4\right]^5 =$

(e) $\left[\left(\frac{1}{2}\right)^3\right]^2 =$

2. Write the following expressions in simplest exponential form.

(a) $(3^5)^6 \times (3^2)^3 =$

(b) $(7^2 \times 8^3)^5 =$

(c) $3^5 \times 8 \times (8^3 \times 3^2)^4 =$

(d) $(2 \times 5^2)^3 \; (5 \times 2^4)^2 =$

D. Quotient of Powers Property

A quotient of powers can be written as a single power if the divisor and dividend have the same base.

EXAMPLE 1: Express the quotient $\frac{3^6}{3^4}$ as a single power.

Solution

$$\frac{3^6}{3^4} = \frac{3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3} \begin{array}{l} \leftarrow \text{6 factors} \\ \leftarrow \text{4 factors} \end{array}$$

$$= \frac{\cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3} \times 3 \times 3}{\cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3}}$$

$$= \frac{3 \times 3}{1}$$

$$= 3^2$$

Note that the same result could have been obtained by retaining the common base 3 and <u>subtracting</u> the exponent of the denominator from the exponent of the numerator. (Note that the numerator has the larger exponent in this case.)

i.e. $\frac{3^6}{3^4} \quad 3^{6-4} = 3^2$

Express each quotient below as a single power.

1. $\frac{(-7)^{18}}{(-7)^7} = (-7)^{18-7} = (-7)^{11}$

2. $\frac{\pi^5}{\pi^3} = \pi^{—\,-\,—} = \pi^{—}$

3. $\frac{\sqrt{2}^9}{\sqrt{2}^2} =$

EXAMPLE 2: Express the quotient $\frac{(-5)^3}{(-5)^6}$ as a single power.

Solution

$$\frac{(-5)^3}{(-5)^6} = \frac{(\cancel{-5})(\cancel{-5})(\cancel{-5})}{(\cancel{-5})(\cancel{-5})(\cancel{-5})(-5)(-5)(-5)} \begin{array}{l} \leftarrow \text{3 factors} \\ \leftarrow \text{6 factors} \end{array}$$

$$= \frac{1}{(-5)(-5)(-5)}$$

$$= \frac{1}{(-5)^3}$$

Note that the same result could have been obtained by dividing one by a power whose base is -5 and whose exponent is the difference between the exponent of the denominator and the exponent of the numerator. (Note that the denominator has the larger exponent in this case.)

i.e. $\frac{(-5)^3}{(-5)^6} = \frac{1}{(-5)^{6-3}} = \frac{1}{(-5)^3}$ Numerator is 1.

Express each quotient below as a single power.

1. $\frac{5}{5^8} = \frac{1}{5^{8-1}} = \frac{1}{5^7}$

2. $\frac{(-6)^5}{(-6)^9} = \frac{1}{(-6)^{\_\_}} = \frac{1}{(-6)^{\_\_}}$

3. $\frac{(\sqrt{3})^3}{(\sqrt{3})^4} = \frac{1}{\_\_\_\_} = \frac{1}{\_\_\_}$

EXAMPLE 3: Simplify the quotient $\frac{2^3}{2^3}$.

Solution

$$\frac{2^3}{2^3} = \frac{\cancel{2} \times \cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2} \times \cancel{2}} = \frac{1}{1} = 1$$

Note that this result is consistent with our previous definitions if we define any real number (except zero) to the zero exponent to be equal to 1.

i.e. $\frac{2^3}{2^3} = 2^{3-3} = 2^0 = 1$

OR

$\frac{2^3}{2^3} = \frac{1}{2^{3-3}} = \frac{1}{2^0} = \frac{1}{1} = 1$

Simplify each quotient below.

1. $\frac{(-3)^8}{(-3)^8} = (-3)^{8-8} = (-3)^{0} = 1$

2. $\frac{17^3}{17^3} =$

3. $\frac{\sqrt{6}^2}{\sqrt{6}^2} =$

DEFINITION OF ZERO EXPONENTS
For any real number a, $a \neq 0$,
$a^0 = 1$

Note: 0^0 is not defined.

Thus, when the base of a power is any non-zero real number and the exponent is zero, the power equals 1. For example,

$$3^0 = 1, \qquad (-17)^0 = 1, \qquad \left(\frac{2}{3}\right)^0 = 1, \qquad \sqrt{2}^0 = 1$$

Fill in the blanks below.

1. $(6.9)^0 =$ ______ 2. $(-9)^0 =$ ______ 3. $9^0 =$ ______

4. $\left(\frac{1}{7}\right)^0 =$ ______ 5. $\left(3\frac{1}{4}\right)^0 =$ ______ 6. $\sqrt{5}^0 =$ ______

The following property tells us how to write a quotient of powers as a single power.

QUOTIENT OF POWERS PROPERTY

For all positive integers m and n and any real number a, $a \neq 0$,

(i) If $m > n$, $\frac{a^m}{a^n} = a^{m-n}$ ← Exponent of numerator is larger.

(ii) If $m < n$, $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$ ← Exponent of denominator is larger.

(iii) If $m = n$, $\frac{a^m}{a^n} = a^0 = 1$ ← Exponents are the same.

When applying the Quotient of Power Property, remember that

1. If the exponent of the numerator is larger, subtract the exponent of the denominator from it.

   e.g. $\frac{5^7}{5^4} = 5^{7-4} = 5^3$

2. If the exponent of the denominator is larger, subtract the exponent of the numerator from it.

   e.g. $\frac{5^4}{5^7} = \frac{1}{5^{7-4}} = \frac{1}{5^3}$

3. If the exponents are the same, the quotient is 1.

   e.g. $\frac{5^4}{5^4} = 1$

The Quotient of Powers Property applies <u>ONLY</u> when the bases of the powers are the <u>SAME</u>. For example, this law cannot be used to write the quotient $\frac{3^5}{5^3}$ as a single power since one power has the base 3 and the other power has the base 5.

Self-correcting Exercise #4

Answers to this exercise may be found on page 48 of this lesson.

1. Where possible, express each of the quotients as a single power.

(a) $\dfrac{(-2)^{12}}{(-2)^4}$ (b) $\dfrac{(\sqrt{2})^7}{(\sqrt{2})^8}$

(c) $\dfrac{3^7}{2^4}$ (d) $\dfrac{4^3}{4^3}$

(e) $\dfrac{\pi^4}{\pi}$ (f) $\dfrac{(-5)^4}{(-5)^9}$

2. Evaluate the following expressions. Do not leave in exponential form.

(a) $(-0.125)^0 =$ (b) $(-5)^0(-5)^2 =$

(c) $\dfrac{2^3}{2^0} =$ (d) $3^2 - 2^0 =$

(e) $\dfrac{\left(\frac{-3}{4}\right)^{16}}{\left(\frac{-3}{4}\right)^{16}} =$ (f) $\dfrac{3}{2^0} + 2^0 =$

3. Write the following expressions in simplest exponential form.

(a) $\dfrac{(7^2)^8}{(7^4)^3} =$

(b) $\dfrac{2^3}{2^5 \times (2^2)^4} =$

(c) $\dfrac{3^3 \times 3^5}{(3^2)^4} =$

E. Power of a Quotient

A power whose base is written as an indicated quotient can be written as a quotient of powers.

EXAMPLE: Express the power $\left(\frac{-2}{7}\right)^4$ as the quotient of powers.

Solution

$$\left(\frac{-2}{7}\right)^4 = \underbrace{\frac{-2}{7} \times \frac{-2}{7} \times \frac{-2}{7} \times \frac{-2}{7}}_{\text{4 factors}}$$

$$= \frac{-2 \times -2 \times -2 \times -2}{7 \times 7 \times 7 \times 7} \begin{array}{l} \longleftarrow \text{4 factors} \\ \longleftarrow \text{4 factors} \end{array}$$

$$= \frac{(-2)^4}{7^4}$$

Note that the same result could have been obtained by applying the exponent 4 to both the numerator and denominator of the quotient $\frac{-2}{7}$.

i.e. $\left(\frac{-2}{7}\right)^4 = \frac{(-2)^4}{7^4}$

In general, a power whose base is written as an indicated quotient can be written as the quotient of powers by applying the exponent to the numerator and denominator of the base.

POWER OF A QUOTIENT PROPERTY
For the positive integer m and any real numbers a and b, $b \neq 0$,
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Self-correcting Exercise #5

Answers to this exercise may be found on page 49 of this lesson.

1. Express each of the following powers as the quotient of powers. (Always use brackets around negative bases.)

(a) $\left(\frac{-5}{2}\right)^5 =$ (b) $\left(\frac{\sqrt{2}}{-3}\right)^7 =$

(c) $\left(\frac{\pi}{2}\right)^8 =$ (d) $\left(\frac{1.5}{7}\right)^2 =$

2. Write the following expressions in simplest exponential form.

(a) $\left(\frac{2}{3}\right)^3 \times 2^5 =$

(b) $\left(\frac{5}{8}\right)^2 \times \left(\frac{8}{5}\right)^7 =$

(c) $\left(\frac{3^2}{2^3}\right)^5 =$

(d) $\frac{4^2 \times 4^4}{(4^2)^3} =$

EXERCISE - POWER PROPERTIES

1. Fill in the blanks.

(a) When you multiply powers with like bases, you keep the common _______________ and _______________ the exponents.

(b) Any non-zero base which is taken to the exponent zero is equal to _____________.

(c) The ______________________________________ Property tells us that $(3^5)^7 = 3^{35}$.

(d) In order to write $(3^2)^4$ with a single exponent, we keep the common base and _________________________ the exponents.

(e) We use the property $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$ to simplify a quotient when the power in the denominator has an exponent that is ____________ than the exponent in the numerator.

(f) The Product of Powers Property and the Quotient of Powers Property only apply to powers that have the same ____________.

(g) According to the _______________ Property, $(3 \times 2)^5 = 3^5 \times 2^5$.

2. In Column I some properties for working with powers are listed. Which property in Column I was used to arrive at each true statement in Column II?

Column I		Column II
A. Product of Powers Property	___	1. $3^0 = 1$
B. Power of a Product Property	___	2. $9^7 \times 9 = 9^8$
C. Power of a Power Property	___	3. $\frac{2^2}{2^4} = \frac{1}{2^2}$
D. Definition of Zero Exponent	B	4. $(3 \times 5)^4 = 3^4 \times 5^4$
E. Quotient of Powers Property	___	5. $(5^2)^3 = 5^6$
F. Power of a Quotient Property	___	6. $5^2 \times 5^3 = 5^5$
	___	7. $\left(\frac{2}{3}\right)^8 = \frac{2^8}{3^8}$
	___	8. $\frac{4^3}{4} = 4^2$
	___	9. $2^7 \times 6^0 = 2^7 \times 1$
	___	10. $\left(\frac{-1}{6}\right)^3 = \frac{(-1)^3}{6^3}$

3. Use the Power Properties to write each of the following expressions as a single power.

(a) $\frac{2^4 \times 2^5}{2^{22}} = \frac{2^{4+5}}{2^{22}} = \frac{2^9}{2^{22}} = \frac{1}{2^{22-9}} = \frac{1}{2^{13}}$

(b) $(7.23)^4(7.23)^3 =$

(c) $5^{19} \times 5^0 =$

(d) $(-2)^6(-2)^3(-2) =$

(e) $\frac{(7^9)^2}{7^5} =$

(f) $\frac{\sqrt{3^4}}{\sqrt{3^{14}}} =$

(g) $\dfrac{5^3 \times 5^5}{5^2} =$

(h) $\dfrac{7^4 \times 7^0 \times 7^4}{7^2 \times 7} =$

(i) $\dfrac{3^{12} \times 3^3}{3} =$

(j) $\dfrac{(-6)(-6)^7}{(-6)^4\,(-6)^5} =$

(k) $\dfrac{(5^3)^8}{(5^2)^5} = \dfrac{5^{3\times 8}}{5^{2\times 5}} =$

(l) $(2^3)^2 \times (2^4)^3 =$

(m) $\dfrac{2^7}{2^5 \times (2^2)^4} =$

4. Evaluate the following expressions. (Do not leave them in exponential form.)

(a) $\dfrac{(-3)^{27}}{(-3)^{24}} = (-3)^{27-24} = (-3)^3 = -3 \times -3 \times -3 = -27$

(b) $3^0 - 4^2 =$

(c) $\dfrac{(-5)^{12}}{(-5)^{15}} =$

(d) $2^0 - 2^2 + 2^4 =$

(e) $\dfrac{(2^5)^4}{(2^3)^5} =$

(f) $\left(\dfrac{-1}{3}\right)^3 + \left(\dfrac{2}{3}\right)^2 =$

5. Write each of the following expressions as the product or quotient of powers.

(a) $(-5 \times 3 \times 8)^3 = \underline{(-5)^3} \times \underline{\quad} \times \underline{\quad}$

(b) $\left(\frac{\pi}{5}\right)^2 =$

(c) $(3 \times 5^4)^6 = 3^6 \times (5^4)^6 =$

(d) $\left[\frac{(-4)^3}{7^2}\right]^5 =$

(e) $\left(\frac{-2}{7}\right)^8 =$

(f) $\left[(-4)^7 (3)^2\right]^4 =$

6. Write each expression in simplest exponential form.

(a) $\dfrac{2 \times 3^4 \times 2^5 \times 3^8}{3^2 \times 2^8}$

$= \dfrac{2^{1+5} \times 3^{4+8}}{3^2 \times 2^8}$

$= \dfrac{2^6 \times 3^{12}}{3^2 \times 2^8}$

$= \dfrac{3^{12-2}}{2^{8-6}}$

$= \dfrac{3^{10}}{2^2}$

(b) $\dfrac{5^2 \times 5^8 \times 3^4}{3^9 \times 3^3 \times 5^4}$

$=$

(c) $\dfrac{(6 \times 5)^{16}}{(5^3)^4 \times (6^5)^2}$

$= \dfrac{6^{16} \times}{5^{12} \times}$

$=$

(d) $\left(\frac{2}{3}\right)^7 \times \left(\frac{3}{2}\right)^4$

$= \dfrac{}{3^7} \times \dfrac{3^4}{}$

$=$

Topic Three: Powers With Negative Exponents

A. Meaning of Negative Exponents

When the exponent of a power is a positive integer, it tells us how many times the base of the power is to be used as a factor. That is, for any positive integer m and any real number "a",

$$\boxed{a^m = \underbrace{a \times a \times a \times \ldots \times a}_{m \text{ FACTORS}}}$$

For example,

$$\left(\frac{2}{3}\right)^4 = \underbrace{\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}}_{4 \text{ FACTORS}} = \frac{16}{81}$$

$$(-\sqrt{3})^5 = \underbrace{-\sqrt{3} \times -\sqrt{3} \times -\sqrt{3} \times -\sqrt{3} \times -\sqrt{3}}_{5 \text{ FACTORS}} = -9\sqrt{3}$$

Similarly, the power $\left(\frac{-3}{5}\right)^9$ tells us that the base _____ is to be used as a factor _____ times.

On pages 14 and 15 of this lesson, we defined a^m to be equal to 1, when m = 0 and "a" is any non-zero real number. That is, when the exponent of a power is zero, the power equals 1.

$$\boxed{a^0 = 1}$$

For example,

$2^0 = 1$, $\left(\frac{3}{4}\right)^0 = 1$, $(-7)^0 = 1$, $(-\sqrt{11})^0 = 1$

What does $(2^3)^0$ equal? _____

Any power that has a base which is a real number and an exponent that is a negative integer is called a NEGATIVE INTEGRAL POWER. The following are all examples of negative integral powers.

3^{-2}, $(-2)^{-5}$, $\left(\frac{1}{2}\right)^{-4}$, π^{-3}, $(-\sqrt{2})^{-7}$

Give examples of three other negative integral powers.

_____, _____, _____

We must define negative integral powers so that our new definition is consistent with previous properties and definitions. That is, we must define a^m, when m is a <u>negative</u> integer, in such a way that the power properties already developed still apply.

Consider the symbol 3^{-2}. If a meaning is to be given to this symbol that is consistent with the power properties already developed, it must be true that:

$$3^{-2} \times 3^2 = 3^{-2+2} \qquad \text{(Product of Powers Property)}$$

$$= 3^0$$

$$= 1 \qquad \text{(Definition of Zero Exponent)}$$

But, if the product of 3^{-2} and 3^2 is one, then these two expressions must be reciprocals (or multiplicative inverses).
Thus, we must define 3^{-2} to be the reciprocal of 3^2.

i.e. 3^{-2} = Reciprocal of 3^2

$$3^{-2} = \frac{1}{3^2}$$

In a similar manner, we define:

2^{-5} to be the reciprocal of 2^5 (i.e. $2^{-5} = \frac{1}{2^5}$)

4^{-1} to be the reciprocal of 4^1 (i.e. $4^{-1} = \frac{1}{4}$)

y^{-6} to be the reciprocal of y^6 (i.e. $y^{-6} = \frac{1}{y^6}$)

In general, a^{-m} is the reciprocal of a^m when $m \in {}^{+}I$. $\left(\text{i.e. } a^{-m} = \frac{1}{a^m}\right)$

DEFINITION OF NEGATIVE EXPONENTS

For any real number "a" ($a \neq 0$) and any positive integer m,

$$a^{-m} = \frac{1}{a^m}$$

Use the definition of negative exponents to evaluate the following expressions.

1. $3^{-5} = \frac{1}{3^{5}} = \frac{1}{243}$

2. $5^{-2} = \frac{1}{5^{\_\_}} = \frac{1}{\_\_\_}$

3. $(-2)^{-3} = \frac{1}{(\quad)^{3}} = \frac{1}{\_\_\_}$

4. $\frac{1}{7^{-2}} = \frac{1}{\frac{1}{7^{2}}} = 1 \div \frac{1}{7^{2}} = 1 \times \frac{7^{2}}{1} = 7^{2} = \_\_\_$

5. $\left(\frac{1}{4}\right)^{-1} = \frac{1}{\frac{1}{4}} = 1 \div \frac{1}{4} = 1 \times \_\_ = \_\_\_$

Self-correcting Exercise #6

Answers to this exercise may be found on page 49 of this lesson.

1. Evaluate the following expressions. (Do not leave them in exponential form.)

(a) $2^{-5} =$

(b) $63^{-1} =$

(c) $(73.2)^{0} =$

(d) $\left(\frac{1}{2}\right)^{-3} =$

(e) $(-3)^{-4} =$

(f) $\frac{1}{5^{-3}} =$

(g) $\frac{3^{0} \times 2^{-5}}{8^{-2}} =$

(h) $(-4)^{3} \times 6^{-2} =$

2. Write with positive exponents. (Leave in exponential form.)

(a) $\dfrac{6^{-2} \times 7^2}{4^{-3}} = \dfrac{\frac{1}{6^2} \times 7^2}{\frac{1}{4^3}} = \dfrac{7^2}{6^2} \div \dfrac{1}{4^3} = \dfrac{7^2}{6^2} \times \dfrac{4^3}{1} = \dfrac{7^2 \times 4^3}{6^2}$

(b) $3^{-15} =$

(c) $\dfrac{1}{(-4)^{-27}} =$

(d) $8^{-16} + 3^{-14} =$

(e) $6^{-12} \times 7^{-9} =$

(f) $\dfrac{8^{-5} \times 9^{-2}}{4^{-13}} =$

(g) $\dfrac{4 \times 5^{-11}}{15^{-9} \times 7} =$

B. <u>Operating With Negative Integral Powers</u>

On pages 7-17 of this lesson, you learned some power properties that can be used when working with positive integral powers. These same properties also apply to negative integral powers.

1. <u>Product of Powers Property</u>

I is the set of integers.

$$\boxed{a^m \cdot a^n = a^{m+n}, \quad \text{where } m, n \in I}$$

In order to multiply powers with like bases, keep the common base and <u>add</u> the exponents.

EXAMPLES:

$3^{-2} \times 3^{-5} = 3^{(-2) + (-5)} = 3^{-7}$ $\left(\text{or } \dfrac{1}{3^7}\right)$

$2^8 \times 2^{-17} = 2^{8 + (-17)} = 2^{-9}$ $\left(\text{or } \dfrac{1}{2^9}\right)$

When multiplying integral powers (powers whose exponents are positive or negative integers or zero), you add the exponents. Thus, you must know the rules for adding two integers. Review these rules on page 23, lesson 3, and then fill in the blanks below.

(a) $16 + (-4) =$ _____ (b) $15 + (-8) =$ _____ (c) $7 + (-9) =$ _____

(d) $-6 + 6 =$ _____ (e) $-13 + 0 =$ _____ (f) $-14 + 23 =$ _____

Now, use the Product of Powers Property to write each product below as a single power.

USE THE RULE FOR ADDING TWO INTEGERS WITH LIKE SIGNS.

(a) $6^{-3} \times 6^{-7} = 6^{\underline{(-3)} + \underline{(-7)}} = 6^{\underline{-10}}$

(b) $5^{-9} \times 5^{5} = 5^{\_\_ + \_\_} = 5^{\_\_}$

(c) $\left(\frac{1}{2}\right)^{-4} \times \left(\frac{1}{2}\right)^{16} = \left(\frac{1}{2}\right)^{\_\_ + \_\_} = \left(\frac{1}{2}\right)^{\_\_}$

(d) $(-\sqrt{2})^{-12} \times (-\sqrt{2})^{-6} = (-\sqrt{2})^{\_\_ + \_\_} = (-\sqrt{2})^{\_\_}$

(e) $(-4)^{8} \times (-4)^{-20} = (-4)^{\_\_\_\_\_} = (-4)^{\_\_}$

2. <u>Power of a Product Property</u>

$$\boxed{(ab)^m = a^m b^m \text{ where } m \in I}$$

A power whose base is an indicated product can be written in an alternate form by applying the exponent to each factor in the base.

EXAMPLES:

$(3 \times 5)^{-12} = 3^{-12} \times 5^{-12} \quad \left(\text{or } \frac{1}{3^{12}} \times \frac{1}{5^{12}}\right)$

$(7 \times -9)^{-6} = 7^{-6} \times (-9)^{-6} \quad \left(\text{or } \frac{1}{7^{6}} \times \frac{1}{(-9)^{6}}\right)$

Use the Power of a Product Property to write each expression below as the product of negative powers. Then, rewrite the product, changing the negative exponents to positive exponents. (i.e. apply the rule that $a^{-m} = \frac{1}{a^{m}}$)

(a) $(-8 \times 5)^{-20} = (-8)^{-20} \times 5^{-20} = \frac{1}{(-8)^{20}} \times \frac{1}{5^{20}}$

(b) $(\sqrt{2} \times \sqrt{5})^{-7} =$ ___ $\times$ ___ $= \frac{1}{\_\_^{7}} \times \frac{1}{\_\_}$

(c) $(2 \times 3 \times -7)^{-15} =$ ___ $\times$ ___ $\times$ ___ $= \frac{1}{\_\_} \times \frac{1}{\_\_} \times \frac{1}{\_\_}$

3. Power of a Power Property

$$\boxed{(a^{m})^{n} = a^{mn} \text{ where } m, n \in I}$$

In order to find the power of a power, keep the same base and multiply the exponents.

EXAMPLES:

$(3^{-4})^{-5} = 3^{-4 \times -5} = 3^{20}$

$\left[(-5)^{3}\right]^{-4} = (-5)^{3 \times -4} = (-5)^{-12}$

When taking the power of a power, you multiply the exponents. Thus, you must know the rules for multiplying two integers. Review these rules on pages 30 to 32, lesson 3, and then fill in the blanks below.

(a) $7 \times -9 =$ _____ (b) $-3 \times -3 =$ _____ (c) $-11 \times 5 =$ _____

(d) $-10 \times -8 =$ _____ (e) $0 \times -7 =$ _____ (f) $-6 \times 4 =$ _____

Now, use the Power of a Power Property to write each power below with a single exponent.

USE THE RULE FOR MULTIPLYING TWO INTEGERS WITH UNLIKE SIGNS.

(a) $(2^{-5})^3 = 2^{\underline{-5} \times \underline{3}} = 2^{\underline{-15}}$

(b) $\left[(-5)^{-7}\right]^{-2} = (-5)^{\_\times\_} = (-5)^{\_}$

(c) $(3^{13})^{-7} = \_\_^{\_\times\_} = \_\_\_$

(d) $(13^{-2})^{-4} =$

(e) $\left[(-\sqrt{2})^{-5}\right]^{-4} =$

4. <u>Quotient of Powers Property</u>

$$\boxed{\frac{a^m}{a^n} = a^{m-n} \quad \text{where } m,\ n \in I}$$

In order to divide powers with like bases, keep the common base and <u>subtract</u> the exponents.

EXAMPLES:

$$\frac{3^2}{3^{-4}} = 3^{2-(-4)} = 3^{2+(+4)} = 3^6$$

$$\frac{(-4)^{-15}}{(-4)^{-8}} = (-4)^{-15-(-8)} = (-4)^{-15+(+8)} = (-4)^{-7}$$

When dividing integral powers, you <u>subtract</u> the exponents. Thus, you must know the rule for subtracting two integers. Review this rule on page 27, lesson 3, and then fill in the blanks below.

(a) $5 - (-12) =$ _____ (b) $(-3) - (-8) =$ _____ (c) $7 - 7 =$ _____

(d) $15 - (-3) =$ _____ (e) $-9 - (-4) =$ _____ (f) $-7 - 3 =$ _____

Now, use the Quotient of Powers Property to write each quotient below as a single power.

(a) $\frac{5^{-6}}{5^3} = 5^{\underline{-6} - \underline{3}} = 5^{\underline{-9}}$

(b) $\frac{2^{14}}{2^{-5}} = 2^{\_\_ - \_\_} = 2^{\_\_}$

(c) $\frac{(-4)^{-6}}{(-4)^{-13}} = (-4)^{\_\_\_\_\_\_\_\_} = (-4)^{\_\_}$

(d) $\frac{\sqrt{2}^{-9}}{\sqrt{2}^{-4}} = (\_\_\_\_\_\_)^{-9-(-4)} = (\_\_)^{\_\_}$

(e) $\frac{3^{-18}}{3^3} =$

5. Power of a Quotient Property

$$\boxed{\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \text{ where } m \in I}$$

A power whose base is a fraction can be written in an alternate form by applying the exponent to both the numerator and denominator of the fraction.

EXAMPLES:

$$\left(\frac{2}{3}\right)^{-8} = \frac{2^{-8}}{3^{-8}} \quad \left(\text{or } \frac{3^8}{2^8}\right)$$

$$\left(\frac{-1}{4}\right)^{-13} = \frac{(-1)^{-13}}{4^{-13}} \quad \left(\text{or } \frac{4^{13}}{(-1)^{13}}\right)$$

$$\frac{1}{\left(\frac{3}{5}\right)^{-7}} = \frac{1}{\frac{3^{-7}}{5^{-7}}} = \frac{5^{-7}}{3^{-7}} \quad \left(\text{or } \frac{3^7}{5^7}\right)$$

Use the Power of a Quotient Property to write each expression below as the quotient of negative powers. Then, rewrite the quotient, changing the negative exponents to positive exponents.

(a) $$\left(\frac{-3}{4}\right)^{-6} = \frac{(-3)^{-6}}{4^{-6}} = \frac{\frac{1}{(-3)^6}}{\frac{1}{4^6}} = \frac{4^6}{(-3)^6}$$

(b) $$\left(\frac{2}{5}\right)^{-3} = \frac{2^{\_\_}}{5^{\_\_}} = \frac{\frac{1}{2^{\_\_}}}{\frac{1}{5^{\_\_}}} = \frac{5^{\_\_}}{2^{\_\_}}$$

(c) $$\left(\frac{\sqrt{2}}{3}\right)^{-8} = \underline{\qquad} = \frac{\frac{1}{\_\_}}{\frac{1}{\_\_}} = \underline{\qquad\quad}$$

Self-correcting Exercise #7

Answers may be found on page 51 of this lesson.

1. Use the Power Properties to write each expression as a single power.

(a) $2^5 \times 2^{-8} = 2^{5+(-8)} = 2^{-3}$

(b) $(-5)^{-7} \times (-5)^{-1} =$

(c) $\frac{3^7}{3^{-2}} =$

(d) $\frac{\sqrt{2}^{-4}}{\sqrt{2}^{9}} =$

(e) $(4^{-6})^7 =$

(f) $\frac{5^{-6} \times 5^{12}}{5^{-4}} =$

(g) $\frac{(7^{-2})^5}{(7^3)^{-4}} =$

2. Write as the product or quotient of powers. Then, write with positive exponents only.

(a) $\left(\frac{-1}{3}\right)^{-9} = \frac{(-1)^{-9}}{3^{-9}} = \frac{\frac{1}{(-1)^9}}{\frac{1}{3^9}} = \frac{3^9}{(-1)^9}$

(b) $(6 \times -5)^{-4} =$

(c) $\left(\frac{5}{7}\right)^{-6} =$

(d) $(3 \times -8 \times 9)^{-5} =$

3. Write in simplest exponential form by applying the Power Properties. (Negative exponents need not be changed to positive exponents.)

(a) $\frac{(3^{-5})^2 \times (3^{-2})^{-3}}{3^8} = \frac{3^{-5 \times 2} \times 3^{-2 \times -3}}{3^8} = \frac{3^{-10} \times 3^6}{3^8} = 3^{-10+6-8} = 3^{-12}$

(b) $\left(\frac{3}{4}\right)^{-7} \times 4^{12} =$

(c) $\frac{(8 \times -3)^{-3}}{8^{-2} \times (-3)^{-5}} =$

(d) $2^{-6} \times 5^7 \times (2^2 \times 8)^{-4} =$

EXERCISE - Integral Exponents

1. Fill in the blanks.

(a) 3^{-1} is defined to be the ______________________ of 3 and is equal to __________.

(b) In order to write $(2^{-3})^4$ with a single exponent, we must keep the base _____, and ____________________ the exponents -3 and _____.

(c) In order to write the product $3^{-5} \times 3^{8}$ as a single power, we must keep the common base _____ and ____________________ the exponents -5 and _____.

(d) $\left(\frac{1}{2}\right)^{-1}$ is equivalent to the integer __________.

(e) $(-3)^{-2}$ is equivalent to the fraction __________.

2. Evaluate the following expressions. (Do not leave them in exponential form.)

(a) $\left(\frac{-3}{4}\right)^{-3} = \frac{(-3)^{-3}}{4^{-3}} = \frac{4^3}{(-3)^3} = \frac{64}{-27} = -2\frac{10}{27}$

(b) $7^{-4} =$

(c) $(-2)^{-5} =$

(d) $\frac{1}{8^{-2}} =$

(e) $\left(\frac{5}{8}\right)^{-1} =$

(f) $\frac{5}{6^{-2}} =$

(g) $2^{-1} + 3^{-1} =$

(In part g, express answer as a single rational number.)

(h) $(-4)^0 + \frac{1}{2^{-6}} =$

(i) $\frac{2^{-3}}{4^{-2}} =$

(j) $\left(\frac{2}{3}\right)^{-4} =$

3. Write each expression with positive exponents only. (Do not evaluate the powers.)

(a) $\frac{3^{-8} \times 4^2}{5^{-2}} = \frac{5^2 \times 4^2}{3^8}$

(b) $5^{-16} =$

(c) $\frac{1}{4^{-7}} =$

(d) $6 \times 8^{-14} =$

(e) $\frac{(-2)^{-11}}{6^2} =$

(f) $\frac{4^{-13}}{3^{-6}} =$

(g) $\frac{4^{-5} \times 3^{-12}}{2^{-8}} =$

(h) $\frac{2^{-13} \times 6^7}{8^{10}} =$

(i) $5^{-12} + 6^{-8} =$

4. Use the Power Properties to simplify the following expressions. Leave answers in exponential form.

(a) $\frac{(-4)^{-10} \times (-4)^{-2}}{(-4)^{-3}} = \frac{(-4)^{-12}}{(-4)^{-3}} = (-4)^{-12-(-3)} = (-4)^{-9}$

(b) $\frac{7^{15}}{7^{-8}} =$

(c) $\frac{\pi^4 \times \pi^2}{\pi^{-3}} =$

(d) $(3^3)^{-8} \times (3^{-2})^{-5} =$

(e) $\left(\frac{2}{3}\right)^{-5} \times 2^6 \times 3^{-8} =$

(f) $\frac{(6^3)^3 \times 6^{-5}}{(6^{-2})^{-2}} =$

5. Use the Power Properties to simplify the following expressions. Then evaluate the expressions. (i.e. do not leave them in exponential form.)

(a) $\frac{3^{-5} \times (3^4)^2}{3^{-2}} = \frac{3^{-5} \times 3^8}{3^{-2}} = \frac{3^3}{3^{-2}} = 3^{3-(-2)} = 3^5 = 243$

(b) $8^{-6} \times 8^4 =$

(c) $\frac{(2 \times -5)^9}{(2^2)^2 \times (-5)^{12}} =$

(d) $\frac{\left[(-3)^2\right]^{-12}}{\left[(-3)^{-4}\right]^6} =$

Topic Four: Using Powers of 10

A. Meaning of Powers of 10

Powers of 10 are expressions that have 10 as a base and any integer as an exponent.

If the exponent is a positive integer, the power of 10 represents a natural number greater than or equal to 10. For example, expressions like 10^1, 10^2, 10^3, 10^4, and so on, represent numbers greater than or equal to ten. In every case, the exponent of the power corresponds to the number of zeros in the numeral it represents.

ONE ZERO

i.e. $10^1 = 10$ $\qquad$ $10^3 = 1000$

$10^2 = 100$ $\qquad$ $10^4 = 10\,000$

and so on.

What numeral does 10^5 represent? __________

If the exponent is a negative integer, the power of 10 represents a fractional number greater than zero and less than or equal to one-tenth. For example, expressions like 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, and so on, represent fractions that are greater than zero and less than or equal to one-tenth. In every case, the absolute value of the exponent of the power corresponds to the number of zeros in the denominator of the fraction.

i.e. $10^{-1} = \frac{1}{10}$ $\qquad$ $10^{-3} = \frac{1}{1000}$

ONE ZERO

$10^{-2} = \frac{1}{100}$ $\qquad$ $10^{-4} = \frac{1}{10\,000}$

4 ZEROS

and so on.

What fraction does 10^{-5} represent? __________

On page 14 of this lesson, you were told that any non-zero base with an exponent of zero is equal to one. Thus,

$$10^0 = 1$$

Evaluate the following powers of 10.

Number must have 6 zeros.

1. $10^6 = \underline{1\ 000\ 000}$ 2. $10^2 = $ ______

3. $10^{-2} = $ ______ 4. $10^4 = $ ______

5. $10^{-1} = $ ______ 6. $10^8 = $ ______

7. $10^0 = $ ______ 8. $10^{-3} = $ ______

Write each of the following numerals as a power of 10.

1. $\underbrace{1\ 000\ 000\ 000}_{9 \text{ zeros}} = \underline{10^9}$ 2. $\frac{1}{1\ 000\ 000} = $ ______

3. $1 = $ ______ 4. $100\ 000\ 000 = $ ______

5. $\frac{1}{1000} = $ ______ 6. $\frac{1}{10\ 000\ 000} = $ ______

B. <u>Using Powers of 10 in the Expanded Form of a Numeral</u>

On page 6, lesson 5, you were introduced to the expanded form of a numeral. You will recall that a numeral is in expanded form when it is written as a sum which shows the place value of each of its digits. For example, the numeral 38 562.243 can be written in expanded form as follows:

$$38\ 562.243 = (3 \times 10\ 000) + (8 \times 1\ 000) + (5 \times 100) + (6 \times 10) + (2 \times 1) + \left(2 \times \frac{1}{10}\right) + \left(4 \times \frac{1}{100}\right) + \left(3 \times \frac{1}{1000}\right)$$

Note that each term in the expanded form contains a numeral that can be expressed as a power of 10. That is, the first term contains the numeral 10 000 which equals 10^4. The second term contains the numeral 1 000 which equals 10^3. Similarly,

$$100 = 10^2, \quad 10 = 10^{-}, \quad \frac{1}{10} = 10^{-}, \quad \frac{1}{100} = 10^{-}, \quad \frac{1}{1000} = 10^{-3}$$

(Fill in the missing exponents above.)

Thus, the expanded form of the numeral 38 562.243 could be written using exponential notation as follows:

10000 has 4 zeros.

$$38\ 562.243 = (3 \times 10^4) + (8 \times 10^3) + (5 \times 10^2) + (6 \times 10^1) + (2 \times 10^0) + (2 \times 10^{-1}) + (4 \times 10^{-2}) + (3 \times 10^{-3})$$

Notice that the exponents of the powers of 10 decrease by one as you go from term to term in the sum.

Write the numeral 130, 678.45 in expanded form, using exponential notation.

since 100,000 has 5 zeros

$$130\ 678.45 = (1 \times 10^5) + (3 \times 10^{\_}) + (0 \times 10^{\_}) + (6 \times \_\_\_) + (7 \times \_\_) + (8 \times \_\_) + (4 \times 10^{-1}) + (5 \times 10^{\_})$$

Self-correcting Exercise #8

Answers may be found on page 52 of this lesson.

1. Each expression below represents a term in the expanded form of a numeral. Evaluate each term.

(a) $(4 \times 10^3) =$ __________ (b) $(7 \times 10^{-1}) =$ 0.7

(c) $(8 \times 10^2) =$ __________ (d) $(5 \times 10^{-3}) =$ __________

(e) $(3 \times 10^7) =$ __________ (f) $(2 \times 10^{-5}) =$ __________

2. Write each numeral as the product of a whole number between 1 and 9 inclusive and a power of 10.

whole number power of ten

(a) $300\ 000 = \underline{3} \times 10^{5}$ (b) $\frac{6}{1000} = 6 \times 10^{\_}$

(c) $500 = \_\_ \times \_\_$ (d) $\frac{1}{10} = \_\_ \times \_\_$

(e) $60\ 000 = \_\_ \times \_\_$ (f) $\frac{7}{100} = \_\_ \times \_\_$

3. Write each numeral in expanded form, using exponential notation.

(a) $695.3 = (6 \times \_\_) + (9 \times \_\_) + (5 \times \_\_) + (3 \times \_\_)$

(b) $7\ 832.05692 = (7 \times \_\_) + (\_\_ \times \_\_) + (\_\_ \times \_\_) + (2 \times 10^0) + (\_\_ \times \_\_) + (5 \times \_\_) + (\_\_ \times \_\_) + (\_\_ \times \_\_) + (2 \times \_\_)$

C. Scientific Notation

Astronomers, physicists, chemists, and engineers often work with numbers that represent either very large or very small quantities. Such numbers are unwieldy for the purposes of recording data and computing. A special way of naming such numbers has been developed.

If a number is expressed as the product of a decimal number greater than or equal to 1 and less than 10, and a power of 10, then the number is said to be expressed in scientific notation. For example, the number 98 000 000 may be written as the product of the decimal number 9.8 (which lies between 1 and 10) and the power of 10, 10^7.

Decimal no. between 1 and 10

Power of 10

i.e. $98\ 000\ 000 = 9.8 \times 10^7$

Check: $9.8 \times 10^7 = 9.8 \times \underbrace{10\ 000\ 000}_{7\ \text{ZEROS}}$

$= 98\ 000\ 000$

Decimal point has been moved 7 places to the right.

Put a check mark beside each of the following expressions that represents a number written in scientific notation. (Remember that the first factor must be a decimal number greater than or equal to 1 and less than 10, and the second factor must be an integral power of 10.

3.4×10^{-5}	✓	13.5×10^2	____
2.0×10^8	____	7.1×10^0	____
0.1×10^{-3}	____	0.79×10^4	____
6.243×10^4	____	0.439×10^{-2}	____
3.6×5^{10}	____	10×10^3	____

Use the following procedure when writing a number in scientific notation.

Step 1: Place the decimal after the first non-zero digit in the number.

Step 2: Determine the power of 10 that this new number must be multiplied by in order to make it equal to the original number.

Study the examples below.

Original Number	Step 1	Step 2	New Number (Scientific Notation)
7983	7.983 Place the decimal after 7, the first non-zero digit.	10^3 To make 7.983 equal to 7983 we must move the decimal <u>three</u> places to the <u>right</u>. This is equivalent to multiplying 7.983 by 1000 or 10^3.	7.983×10^3 Check: 7.983×10^3 $= 7.983 \times 1000$ $= 7983$
0.0687	6.87 Place the decimal after 6, the first non-zero digit.	10^{-2} To make 6.87 equal to 0.0687 we must move the decimal <u>two</u> places to the <u>left</u>. This is equivalent to multiplying 6.87 by $\frac{1}{100}$ or 10^{-2}.	6.87×10^{-2} Check: 6.87×10^{-2} $= 6.87 \times \frac{1}{100}$ $= 0.0687$

<u>Self-correcting Exercise #9</u>

Answers may be found on page 53 of this lesson.

1. Express each numeral in scientific notation by filling in the blanks.

(a) 142 000 000

(i) If we place the decimal point after the first non-zero digit, we obtain the decimal number _____ which lies between 1 and 10.

(ii) To make this decimal number equal to the original number 142 000 000 we must move the decimal point _____ places to the __________. This is equivalent to multiplying the decimal number by ____________________ or $10^{\_}$.

(iii) 142 000 000 can be written in scientific notation as ___ $\times\ 10^{\_}$.

(b) 0.000 053 7

(i) If we place the decimal point after the first non-zero digit, we obtain the decimal number __________ which lies between 1 and 10.

(ii) To make this decimal number equal to the original number, 0.000 053 7, we must move the decimal point _____ places to the ______________. This is equivalent to multiplying the decimal number by _____ or 10^{-}.

(iii) 0.000 053 7 can be written in scientific notation as ___ $\times\ 10^{-}$.

2. Write each number in scientific notation and then check your work.

	Number	Scientific Notation	Check
(a)	520	5.2×10^2	$5.2 \times 100 = 520$
(b)	0.032		
(c)	125 000		
(d)	0.000 000 3		
(e)	32 000 000		
(f)	0.005 69		
(g)	7 000 000 000		
(h)	0.973		
(i)	9 500		

D. <u>Using Scientific Notation in Calculations</u>

Computation with very small or very large numbers can be simplified considerably if the numbers are written in scientific notation and the power properties applied.

EXAMPLE: $\dfrac{85\ 000\ 000 \times 960\ 000}{0.004} = \boxed{}$

<u>Solution</u>

First, express all the numbers in scientific notation.

$$85\ 000\ 000 = 8.5 \times 10^7$$

$$960\ 000 = 9.6 \times 10^5$$

$$0.004 = 4.0 \times 10^{-3}$$

Write the given expression using scientific notation.

$$\frac{(8.5 \times 10^7)(9.6 \times 10^5)}{(4.0 \times 10^{-3})}$$

Use the commutative and associative properties of multiplication to group the decimal numbers together and the powers of 10 together.

$$= \frac{8.5 \times \overset{2.4}{\cancel{9.6}}}{\underset{1}{\cancel{4.0}}} \times \frac{10^7 \times 10^5}{10^{-3}}$$

Use the power properties to simplify the powers of 10.

$$= (8.5 \times 2.4) \times 10^{7+5-(-3)}$$

$$= 20.4 \times 10^{15}$$

Write the final answer in scientific notation.

$$= (2.04 \times 10^1) \times 10^{15}$$

Use the Product of Powers Property to find $10^1 \times 10^{15}$.

$$= \underline{2.04 \times 10^{16}}$$

Self-correcting Exercise #10

Answers may be found on page 54 of this lesson.

1. Use the power properties to compute the following. Write your final answers in scientific notation.

(a) $\dfrac{9.3 \times 10^3}{3.0 \times 10^{-4}} =$

(b) $\dfrac{(7.5 \times 10^{-5})(8.4 \times 10^3)}{1.5 \times 10^6} =$

2. Write each number in scientific notation and then compute. Express final answers without using scientific notation.

(a) $64\ 000\ 000 \times 0.000\ 000\ 073$

$= (\_\_ \times 10^{\_})(\_\_ \times 10^{\_})$

$= (\_\_ \times \_\_)(10^{\_} \times 10^{\_})$

$= \_\_\_\_ \times 10^{\_}$

$= \_\_\_\_\_\_$

(b) $\dfrac{0.000\ 000\ 009\ 9}{0.000\ 003}$

$= \dfrac{\_\_ \times 10^{\_}}{\_\_ \times 10^{\_}}$

$= \_\_ \times 10^{\_}$

$= \_\_\_\_\_\_$

EXERCISE - Using Powers of 10

1. Fill in the blanks.

(a) A scientist estimated that the total number of atoms in the universe is 3×10^{74}. This number written in standard form would have ________ zeros.

(b) The numeral 40 000 000 000 is read, "________ ________." It could be written in scientific notation as $\_\_ \times 10^{\_}$.

(c) The radius in centimeters of the nucleus of an atom is 0.000 000 000 003. This number can be written in scientific notation as ____________.

(d) Red light has a wavelength of 6.7×10^{-5} cm. This number can be written as the decimal ______________.

2. Decide whether each of the following statements is true or false.

		True or False?
(a)	$10^5 = 10 \times 10 \times 10 \times 10 \times 10$	________
(b)	$10^3 = 30$	________
(c)	$10^{-2} = \frac{1}{100}$	________
(d)	$10^0 = 0$	________
(e)	$10^4 = 10^3 \times 10$	________
(f)	$10^4 = 10^2 \times 10^2$	________
(g)	$10^3 = 10^6 \div 10^2$	________
(h)	$10^{-5} = 10^3 \times 10^{-2}$	________
(i)	$10^9 = 10^6 \div 10^{-3}$	________
(j)	$10^6 = 100\ 000$	________
(k)	$10^{-3} = \frac{1}{-10^3}$	________

3. Express each decimal as a fraction and then as a power of 10.

	Decimal	Fraction	Power of 10
(a)	0.1	$\frac{1}{10}$	10^{-1}
(b)	0.0001	________	________
(c)	0.01	________	________
(d)	0.000 000 01	________	________
(e)	0.000 001	________	________

4. Write each numeral in expanded form using exponential notation.

(a) 36.405 7 = (__ × __) + (__ × __) + (__ × __)

+ (0×10^{-2}) + (__ × __) + (__ × __)

(b) 68 432.9 = (__ × __) + (__ × __) + (__ × __)

+ (__ × __) + (__ × __) + (__ × __)

5. Use the Product of Powers Property and the Quotient of Powers Property to write the following expressions as single powers.

(a) $\dfrac{10^7 \times 10^{-9}}{10^{-4}} = \dfrac{10^{7+(-9)}}{10^{-4}} = \dfrac{10^{-2}}{10^{-4}} = 10^{(-2)-(-4)} = 10^{-2+4} = 10^2$

(b) $10^{-5} \times 10^3 =$

(c) $10^{-7} \times 10^{-2} =$

(d) $\dfrac{10^{-6}}{10^{-4}} =$

(e) $\dfrac{10^8}{10^{-5}} =$

(f) $\dfrac{10^{-2} \times 10^{-4}}{10^{-7}} =$

(g) $\dfrac{10^{-8} \times 10^3 \times 10^2}{10^{-3} \times 10^6} =$

6. Look at each circled digit and state what number it represents. Then write the value of this digit in scientific notation.

	Value of Circled Digit	Value in Scientific Notation
(a) 6789.4372	0.007	7.0×10^{-3}
(b) 73 052.6		
(c) 5.904 562		
(d) 95 000 623		
(e) 23.864		
(f) 3.052 68		

7. Give the decimal number that corresponds to each expanded form. (Zero terms have been omitted in the expanded form.)

(a) $(5 \times 10^3) + (2 \times 10^0) + (3 \times 10^{-4})$ 5 002·0003

(b) $(4 \times 10^6) + (1 \times 10^2) + (5 \times 10^{-2})$ ____

(c) $(8 \times 10^0) + (7 \times 10^{-1}) + (5 \times 10^{-3}) + (6 \times 10^{-4})$ ____

(d) $(4 \times 10^{-3}) + (5 \times 10^{-5})$ ____

(e) $(8 \times 10^5) + (3 \times 10^3) + (6 \times 10^{-1}) + (2 \times 10^{-3})$ ____

(f) $(7 \times 10^2) + (5 \times 10^1) + (3 \times 10^{-1}) + (2 \times 10^{-2})$ ____

8. Write each number without using scientific notation.

Move decimal point 8 places to the left.

(a) $5.9 \times 10^{-8} = 0.000\ 000\ 059$

(b) $1.4 \times 10^3 =$ ____

(c) $7.77 \times 10^5 =$ ____

(d) $4 \times 10^9 =$ ____

(e) $6.6 \times 10^{-6} =$ ____

(f) $1.09 \times 10^6 =$ ____

(g) $6 \times 10^{-3} =$ ____

(h) $2 \times 10^4 =$ ____

(i) $1.2 \times 10^8 =$ ____

(j) $5 \times 10^{-7} =$ ____

9. Write each number in scientific notation.

(a) 34 000 000 = ____

(b) 0.000 056 = ____

(c) 7 235 000 000 = ____

(d) 0.000 000 006 = ____

10. Compute. Give the answers in scientific notation.

(a) $(3 \times 10^6)(4 \times 10^{-2})$

$= (3 \times 4)(10^6 \times 10^{-2})$

$= 12 \times 10^4$

$= (1.2 \times 10^1) \times 10^4$

$= 1.2 \times 10^5$

(b) $\dfrac{1.6 \times 10^2}{8 \times 10^{-2}}$

$=$

(c) $\dfrac{(4 \times 10^{-9})(9 \times 10^5)}{6 \times 10^{-3}}$

$=$

(d) $\dfrac{(1.64 \times 10^{-4})(2 \times 10^{13})}{4 \times 10^{7}}$

$=$

11. Write each number in scientific notation and then compute. Express final answers without using scientific notation.

(a) 0.000 671 x 43 000 000

$= (6.71 \times 10^{-4})(\_\_ \times 10^{\_})$

$= (6.71 \times \_\_\_)(10^{-4} \times 10^{\_})$

$= \_\_\_\_\_ \times 10^{\_}$

$= 28\ 853$

(b) 580 000 × 600 000 000

$=$

(c) $\dfrac{183\ 000}{0.000\ 000\ 3}$

$= \dfrac{\_\_ \times 10^{\_}}{\_\_ \times 10^{\_}}$

$= \_\_\_\_\_ \times 10^{\_}$

$= \_\_\_\_\_\_\_\_$

(d) $\dfrac{0.000\ 002\ 4 \times 55\ 000}{6000}$

$= \dfrac{(\_\_ \times 10^{\_})(\_\_ \times 10^{\_})}{\_\_ \times 10^{\_}}$

$= \dfrac{\_\_ \times \_\_}{\_\_} \times \dfrac{10^{\_} \times 10^{\_}}{10^{\_}}$

$= \_\_ \times 10^{\_\_\_\_}$

$= \_\_ \times 10^{\_}$

$= \_\_\_\_\_\_\_\_$

Key to Self-correcting Exercises in Lesson 7

Exercise #1, page 8

1. (a) $3^7 \times 3^3 = 3^{7+3} = 3^{10}$

(b) $(-2)^5 \times (-2)^{12} = (-2)^{5+12} = (-2)^{17}$

(c) $3^5 \times 5^3$; Bases not the same.

(d) $\left(\frac{1}{2}\right)^2 \left(\frac{3}{5}\right)^3$; Bases not the same.

(e) $\pi^2 \times \pi = \pi^{2+1} = \pi^3$

(f) $\sqrt{3} \times (\sqrt{3})^2 \times (\sqrt{3})^3 = (\sqrt{3})^{1+2+3} = (\sqrt{3})^6$

(g) $\left(\frac{3}{8}\right)^7 \times \left(\frac{3}{8}\right)^3 \times \left(\frac{3}{8}\right)^6 = \left(\frac{3}{8}\right)^{7+3+6} = \left(\frac{3}{8}\right)^{16}$

(h) $2^3 \times 3^2$; Bases not the same.

Exercise #2, page 10

1. (a) $(5 \times 6)^9 = 5^9 \times 6^9$

(b) $(7 \times 8 \times 9)^{12} = 7^{12} \times 8^{12} \times 9^{12}$

(c) $(-2 \times \pi)^3 = (-2)^3 \times \pi^3$

(d) $\left(\frac{1}{2} \times \frac{3}{5} \times \frac{-7}{8}\right)^5 = \left(\frac{1}{2}\right)^5 \left(\frac{3}{5}\right)^5 \left(\frac{-7}{8}\right)^5$

(e) $(-\sqrt{3} \times \sqrt{5} \times \sqrt{7} \times \sqrt{10})^2 = (-\sqrt{3})^2 \times \sqrt{5}^2 \times \sqrt{7}^2 \times \sqrt{10}^2$

(f) $\left(-3 \times \frac{7}{11} \times 0.2\right)^6 = (-3)^6 \left(\frac{7}{11}\right)^6 (0.2)^6$

Exercise #3, page 12

1. (a) $(0.5^2)^{14} = (0.5)^{2 \times 14} = (0.5)^{28}$

(b) $\left[(-2)^4\right]^2 = (-2)^{4 \times 2} = (-2)^8$

(c) $\left[(7^5)^6\right]^2 = 7^{5 \times 6 \times 2} = 7^{60}$

(d) $\left[(-\sqrt{2})^4\right]^5 = (-\sqrt{2})^{4 \times 5} = (-\sqrt{2})^{20}$

(e) $\left[\left(\frac{1}{2}\right)^3\right]^2 = \left(\frac{1}{2}\right)^{3 \times 2} = \left(\frac{1}{2}\right)^6$

multiply exponents add exponents

2. (a) $(3^5)^6 \times (3^2)^3 = 3^{5\times 6} \times 3^{2\times 3} = 3^{30} \times 3^6 = 3^{30+6} = 3^{36}$

(b) $(7^2 \times 8^3)^5 = (7^2)^5 \times (8^3)^5 = 7^{2\times 5} \times 8^{3\times 5} = 7^{10} \times 8^{15}$

(c) $3^5 \times 8 \times (8^3 \times 3^2)^4 = 3^5 \times 8 \times (8^3)^4 \times (3^2)^4 = 3^5 \times 8 \times 8^{12} \times 3^8 = 3^{13} \times 8^{13}$

(d) $(2 \times 5^2)^3 (5 \times 2^4)^2 = 2^3 \times (5^2)^3 \times 5^2 \times (2^4)^2 = 2^3 \times 5^6 \times 5^2 \times 2^8 = 2^{11} \times 5^8$

Exercise #4, page 16

1. (a) $\dfrac{(-2)^{12}}{(-2)^4} = (-2)^{12-4} = (-2)^8$

(b) $\dfrac{(\sqrt{2})^7}{(\sqrt{2})^8} = \dfrac{1}{(\sqrt{2})^{8-7}} = \dfrac{1}{\sqrt{2}}$

(c) $\left.\begin{array}{l}\dfrac{3^7}{2^4}\end{array}\right\}$ Bases not the same.

(d) $\dfrac{4^3}{4^3} = 4^{3-3} = 4^0 = 1$

(e) $\dfrac{\pi^4}{\pi} = \pi^{4-1} = \pi^3$

(f) $\dfrac{(-5)^4}{(-5)^9} = \dfrac{1}{(-5)^{9-4}} = \dfrac{1}{(-5)^5}$

2. (a) $(-0.125)^0 = 1$

(b) $(-5)^0(-5)^2 = 1 \times (-5)^2 = (-5)(-5) = 25$

(c) $\dfrac{2^3}{2^0} = \dfrac{2^3}{1} = 2 \times 2 \times 2 = 8$

(d) $3^2 - 2^0 = 9 - 1 = 8$

(e) $\dfrac{\left(\frac{-3}{4}\right)^{16}}{\left(\frac{-3}{4}\right)^{16}} = \left(\dfrac{-3}{4}\right)^{16-16} = \left(\dfrac{-3}{4}\right)^0 = 1$

(f) $\dfrac{3}{2^0} + 2^0 = \dfrac{3}{1} + 1 = 3 + 1 = 4$

multiply exponents subtract exponents

3. (a) $\dfrac{(7^2)^8}{(7^4)^3} = \dfrac{7^{2\times 8}}{7^{4\times 3}} = \dfrac{7^{16}}{7^{12}} = 7^{16-12} = 7^4$

(b) $\dfrac{2^3}{2^5 \times (2^2)^4} = \dfrac{2^3}{2^5 \times 2^{2\times 4}} = \dfrac{2^3}{2^5 \times 2^8} = \dfrac{2^3}{2^{5+8}} = \dfrac{2^3}{2^{13}} = \dfrac{1}{2^{13-3}} = \dfrac{1}{2^{10}}$

add exponents subtract exponents

add exponents

(c) $\dfrac{3^3 \times 3^5}{(3^2)^4} = \dfrac{3^{3+5}}{3^{2\times 4}} = \dfrac{3^8}{3^8} = 3^{8-8} = 3^0 = 1$

multiply exponents

Exercise #5, page 17

1. (a) $\left(\frac{-5}{2}\right)^5 = \frac{(-5)^5}{2^5}$ (b) $\left(\frac{\sqrt{2}}{-3}\right)^7 = \frac{\sqrt{2}^7}{(-3)^7}$

(c) $\left(\frac{\pi}{2}\right)^8 = \frac{\pi^8}{2^8}$ (d) $\left(\frac{1.5}{7}\right)^2 = \frac{(1.5)^2}{7^2}$

2. (a) $\left(\frac{2}{3}\right)^3 \times 2^5 = \frac{2^3}{3^3} \times 2^5 = \frac{2^{3+5}}{3^3} = \frac{2^8}{3^3}$

(b) $\left(\frac{5}{8}\right)^2 \times \left(\frac{8}{5}\right)^7 = \frac{5^2}{8^2} \times \frac{8^7}{5^7} = \frac{5^2}{5^7} \times \frac{8^7}{8^2} = \frac{1}{5^{7-2}} \times 8^{7-2} = \frac{8^5}{5^5}$

(c) $\left(\frac{3^2}{2^3}\right)^5 = \frac{(3^2)^5}{(2^3)^5} = \frac{3^{2\times 5}}{2^{3\times 5}} = \frac{3^{10}}{2^{15}}$

(d) $\frac{4^2 \times 4^4}{(4^2)^3} = \frac{4^{2+4}}{4^{2\times 3}} = \frac{4^6}{4^6} = 1$

Exercise #6, page 24

1. (a) $2^{-5} = \frac{1}{2^5} = \frac{1}{32}$

(b) $63^{-1} = \frac{1}{63}$

(c) $(73.2)^0 = 1$

(d) $\left(\frac{1}{2}\right)^{-3} = \frac{1}{\left(\frac{1}{2}\right)^3} = \frac{1}{\frac{1}{8}} = 1 \div \frac{1}{8} = 1 \times \frac{8}{1} = 8$

(e) $(-3)^{-4} = \frac{1}{(-3)^4} = \frac{1}{-3 \times -3 \times -3 \times -3} = \frac{1}{81}$

(f) $\frac{1}{5^{-3}} = \frac{1}{\frac{1}{5^3}} = 1 \div \frac{1}{125} = 1 \times \frac{125}{1} = 125$

(g) $$\frac{3^0 \times 2^{-5}}{8^{-2}} = \frac{1 \times \frac{1}{2^5}}{\frac{1}{8^2}} = \frac{\frac{1}{32}}{\frac{1}{64}} = \frac{1}{32} \times \frac{64}{1} = 2$$

(h) $$(-4)^3 \times 6^{-2} = (-4 \times -4 \times -4) \times \frac{1}{6^2} = \frac{-64}{36} = \frac{-16}{9}$$

2. (a) $$\frac{6^{-2} \times 7^2}{4^{-3}} = \frac{\frac{1}{6^2} \times 7^2}{\frac{1}{4^3}} = \frac{7^2}{6^2} \div \frac{1}{4^3} = \frac{7^2}{6^2} \times \frac{4^3}{1} = \frac{7^2 \times 4^3}{6^2}$$

(b) $$3^{-15} = \frac{1}{3^{15}}$$

(c) $$\frac{1}{(-4)^{-27}} = \frac{1}{\frac{1}{(-4)^{27}}} = (-4)^{27}$$

(d) $$8^{-16} + 3^{-14} = \frac{1}{8^{16}} + \frac{1}{3^{14}}$$

(e) $$6^{-12} \times 7^{-9} = \frac{1}{6^{12}} \times \frac{1}{7^9}$$

(f) $$\frac{8^{-5} \times 9^{-2}}{4^{-13}} = \frac{\frac{1}{8^5} \times \frac{1}{9^2}}{\frac{1}{4^{13}}} = \frac{1}{8^5 \times 9^2} \div \frac{1}{4^{13}} = \frac{1}{8^5 \times 9^2} \times \frac{4^{13}}{1} = \frac{4^{13}}{8^5 \times 9^2}$$

(g) $$\frac{4 \times 5^{-11}}{15^{-9} \times 7} = \frac{4 \times \frac{1}{5^{11}}}{\frac{1}{15^9} \times 7} = \frac{\frac{4}{5^{11}}}{\frac{7}{15^9}} = \frac{4}{5^{11}} \times \frac{15^9}{7} = \frac{4 \times 15^9}{5^{11} \times 7}$$

Exercise #7, page 30

1. (a) $2^5 \times 2^{-8}$ $\quad 2^{5+(-8)} = 2^{-3}$ add the exponents

(b) $(-5)^{-7} \times (-5)^{-1} = (-5)^{(-7)+(-1)} = (-5)^{-8}$

(c) $\frac{3^7}{3^{-2}} = 3^{7-(-2)} = 3^{7+2} = 3^9$ subtract the exponents

(d) $\frac{\sqrt{2}^{-4}}{\sqrt{2}^9} = \sqrt{2}^{-4-9} = \sqrt{2}^{-13}$

multiply the exponents

(e) $(4^{-6})^7 = 4^{-6 \times 7} = 4^{-42}$

(f) $\frac{5^{-6} \times 5^{12}}{5^{-4}} = \frac{5^{-6+12}}{5^{-4}} = \frac{5^6}{5^{-4}} = 5^{6-(-4)} = 5^{6+4} = 5^{10}$

(g) $\frac{(7^{-2})^5}{(7^3)^{-4}} = \frac{7^{-2 \times 5}}{7^{3 \times -4}} = \frac{7^{-10}}{7^{-12}} = 7^{-10-(-12)} = 7^{-10+12} = 7^2$

2. (a) $\left(\frac{-1}{3}\right)^{-9} = \frac{(-1)^{-9}}{(3)^{-9}} = \frac{\frac{1}{(-1)^9}}{\frac{1}{3^9}} = \frac{3^9}{(-1)^9}$

(b) $(6 \times -5)^{-4} = 6^{-4} \times (-5)^{-4} = \frac{1}{6^4} \times \frac{1}{(-5)^4}$

(c) $\left(\frac{5}{7}\right)^{-6} = \frac{5^{-6}}{7^{-6}} = \frac{\frac{1}{5^6}}{\frac{1}{7^6}} = \frac{7^6}{5^6}$

(d) $(3 \times -8 \times 9)^{-5} = 3^{-5} \times (-8)^{-5} \times 9^{-5} = \frac{1}{3^5} \times \frac{1}{(-8)^5} \times \frac{1}{9^5}$

Multiply exponents

3. (a) $\dfrac{(3^{-5})^2 \times (3^{-2})^{-3}}{3^8} = \dfrac{3^{-5\times 2} \times 3^{-2\times -3}}{3^8} = \dfrac{3^{-10} \times 3^6}{3^8} = 3^{-10+6-8} = 3^{-12}$

(b) $\left(\dfrac{3}{4}\right)^{-7} \times 4^{12} = \dfrac{3^{-7}}{4^{-7}} \times 4^{12} = 3^{-7} \times 4^{12-(-7)} = 3^{-7} \times 4^{19}$

(c) $\dfrac{(8 \times -3)^{-3}}{8^{-2} \times (-3)^{-5}} = \dfrac{8^{-3}}{8^{-2}} \times \dfrac{(-3)^{-3}}{(-3)^{-5}} = 8^{-3-(-2)} \times (-3)^{-3-(-5)} = 8^{-1} \times (-3)^2$

(d) $2^{-6} \times 5^7 \times (2^2 \times 8)^{-4} = 2^{-6} \times 5^7 \times (2^2)^{-4} \times 8^{-4} = 2^{-6} \times 5^7 \times 2^{-8} \times 8^{-4}$

$= 2^{-14} \times 5^7 \times 8^{-4}$

Exercise #8, page 37

1. (a) $4 \times 10^3 = \underline{4000}$ (Move decimal point $\underline{3}$ places to the $\underline{\text{right}}$ since exponent is 3.)

(b) $7 \times 10^{-1} = \underline{0.7}$ (Move decimal point $\underline{1}$ place to the $\underline{\text{left}}$ since exponent is -1.)

(c) $8 \times 10^2 = \underline{800}$ (Move decimal point $\underline{2}$ places to the $\underline{\text{right}}$ since exponent is 2.)

(d) $5 \times 10^{-3} = \underline{0.005}$ (Move decimal point $\underline{3}$ places to the $\underline{\text{left}}$ since exponent is -3.)

(e) $3 \times 10^7 = \underline{30\ 000\ 000}$ (Move decimal point $\underline{7}$ places to the $\underline{\text{right}}$ since exponent is 7.)

(f) $2 \times 10^{-5} = \underline{0.000\ 02}$ (Move decimal point $\underline{5}$ places to the $\underline{\text{left}}$ since exponent is -5.)

(Exponent corresponds to no. of zeros)

2. (a) $300\ 000 = 3 \times 10^5$ (b) $\dfrac{6}{1000} = 6 \times 10^{-3}$

(c) $500 = 5 \times 10^2$ (d) $\dfrac{1}{10} = 1 \times 10^{-1}$

(e) $60\ 000 = 6 \times 10^4$ (f) $\dfrac{7}{100} = 7 \times 10^{-2}$

3. (a) $695.3 = (6 \times 10^2) + (9 \times 10^1) + (5 \times 10^0) + (3 \times 10^{-1})$

(b) $7\ 832.056\ 92 = (7 \times 10^3) + (8 \times 10^2) + (3 \times 10^1) + (2 \times 10^0)$
$+ (0 \times 10^{-1}) + (5 \times 10^{-2}) + (6 \times 10^{-3}) + (9 \times 10^{-4}) + (2 \times 10^{-5})$

Exercise #9, page 39

1. (a) 142 000 000

(i) If we place the decimal point after the first non-zero digit, we obtain the decimal number 1.42 which lies between 1 and 10.

(ii) To make this decimal number equal to the original number 142 000 000, we must move the decimal point 8 places to the right. This is equivalent to multiplying the decimal number by 100 000 000 or 10^8.

(iii) 142 000 000 can be written in scientific notation as 1.42×10^8.

(b) 0.000 053 7

(i) If we place the decimal point after the first non-zero digit, we obtain the decimal number 5.37 which lies between 1 and 10.

(ii) To make this decimal number equal to the original number 0.000 053 7, we must move the decimal point 5 places to the left. This is equivalent to multiplying the decimal number by $\frac{1}{100\ 000}$ or 10^{-5}.

(iii) 0.000 053 7 can be written in scientific notation as 5.37×10^{-5}.

	Number	Scientific Notation	Check
2. (a)	520	5.2×10^2	$5.2 \times 100 = 520$
(b)	0.032	3.2×10^{-2}	$3.2 \times \frac{1}{100} = 0.032$
(c)	125 000	1.25×10^5	$1.25 \times 100\ 000 = 125\ 000$
(d)	0.000 000 3	3.0×10^{-7}	$3 \times \frac{1}{10\ 000\ 000} = 0.000\ 000\ 3$
(e)	32 000 000	3.2×10^7	$3.2 \times 10\ 000\ 000 = 32\ 000\ 000$
(f)	0.005 69	5.69×10^{-3}	$5.69 \times \frac{1}{1000} = 0.005\ 69$

	Number	Scientific Notation	Check
(g)	7 000 000 000	7.0×10^9	$7.0 \times 1\ 000\ 000\ 000 = 7\ 000\ 000\ 00$
(h)	0.973	9.73×10^{-1}	$9.73 \times \frac{1}{10} = 0.973$
(i)	9 500	9.5×10^3	$9.5 \times 1000 = 9\ 500$

Exercise #10, page 42

1. (a) $\frac{9.3 \times 10^3}{3.0 \times 10^{-4}} = \frac{9.3}{3} \times 10^{3-(-4)} = \underline{3.1 \times 10^7}$

(b) $\frac{(7.5 \times 10^{-5})(8.4 \times 10^3)}{1.5 \times 10^6} = \frac{\overset{5}{\cancel{7.5}} \times 8.4}{\underset{1}{\cancel{1.5}}} \times 10^{-5+3-6} = 42 \times 10^{-8}$

$= (4.2 \times 10^1) \times 10^{-8}$

$= 4.2 \times 10^{-7}$

2. (a) $64\ 000\ 000 \times 0.000\ 000\ 073$

$= (6.4 \times 10^7)(7.3 \times 10^{-8})$

$= (6.4 \times 7.3)(10^7 \times 10^{-8})$

$= 46.72 \times 10^{-1}$

$= \underline{4.672}$

(b) $\frac{0.000\ 000\ 009\ 9}{0.000\ 003}$

$= \frac{9.9 \times 10^{-9}}{3.0 \times 10^{-6}}$

$= 3.3 \times 10^{-9-(-6)}$

$= 3.3 \times 10^{-3}$

$= \underline{0.0033}$

End of Lesson 7

Lesson 8

Variable Expressions

Basic Algebra and Geometry

VARIABLE EXPRESSIONS

Topic One: What is a Variable Expression?

In Lessons 1-6 of this course, you were introduced to the sets of numbers N, W, I, Q, and R and were shown how to operate with these numbers. In this lesson, you will be asked to operate with expressions that not only involve elements of set R, but also involve letter symbols called variables. A VARIABLE is a letter symbol which may represent any of the elements of a specified number set. For example, letter symbols like a, b, x, y, and so on, are often used as variables in mathematical expressions.

Mathematical phrases that involve numbers, variables, and symbols for arithmetic operations are called VARIABLE EXPRESSIONS or ALGEBRAIC EXPRESSIONS. For example, $5x^2y - 3$ is a variable expression involving the real numbers 5 and 3, the variables x and y, and the operations of multiplication, squaring, and subtraction.

For each variable expression given below, state what real numbers, variables, and arithmetic operations are involved.

Variable Expression	Real Numbers	Variables	Arithmetic Operations
$\frac{9\sqrt{x + y}}{8z}$	9, 8	x, y, z	multiplication, addition square root, division
$\frac{2n^2}{3n} - 5p$			
$\frac{3(a - b)}{7cd^3}$			
$9(a - 2b + 12c)$			
$5y - \sqrt{x + 2}$			

A variable expression tells us how to operate with the numbers and variables involved. If the variables are replaced by real numbers, the expression can be evaluated. For example, the variable expression $\frac{x + 5}{2}$ instructs us to increase a number by 5 and divide the result by 2. If we replace x by a particular value, then we can perform the operations of adding 5 and dividing by 2.

What instructions are given by the following variable expressions?

1. $\frac{x}{3} + 4$ instructs us to ____________________ a number by 3 and increase the result by 4.

2. $7(x + 1)$ instructs us to ____________________ a number by 1 and ____________________ the result by 7.

3. $-8y^2$ instructs us to ____________________ a number and multiply the result by __________.

4. $\sqrt{z - 5}$ instructs us to ____________________ 5 from a number and then take the principal ____________________ root of the result.

5. $-4y - 7$ instructs us to ____________________ a number by -4 and then subtract __________ from the result.

A. Evaluating Variable Expressions

A variable expression tells us what operations are to be performed on the numbers and variables involved, but it does not take on a numerical value until each variable is replaced by a specific value.

The set of values that may serve as replacements for a variable is called the DOMAIN of the variable. The individual members of the domain are called the VALUES OF THE VARIABLE.

If an algebraic expression involves only one variable and this variable is assigned a specific value from its domain, all the indicated operations can be performed and a numerical value can be determined for the expression.

EXAMPLE: The domain of the variable y is $\{-2, 0, 5\}$. Find a numerical value for the expression $y^2 - 2y - 6$ for each value of the variable.

<u>Solution</u>

(a) When $y = -2$,

$$y^2 - 2y - 6 = (-2)^2 - 2(-2) - 6 = 4 + 4 - 6 = 8 - 6 = \_\_\_\_$$

(b) When $y = 0$,

$$y^2 - 2y - 6 = (0)^2 - 2(0) - 6 = 0 - 0 - 6 = \_\_\_\_$$

(c) When $y = 5$,

$$y^2 - 2y - 6 = (5)^2 - 2(5) - 6 = 25 - 10 - 6 = 15 - \_\_\_\_ = \_\_\_\_$$

Some algebraic expressions involve more than one variable. Each variable involved must be replaced by a specific value from its domain before the expression can be evaluated.

EXAMPLE: Evaluate $\frac{1}{2}x - 5y + 4z^2$ when $x = 18$, $y = 4$ and $z = 2$.

<u>Solution</u>

In the variable expression $\frac{1}{2}x - 5y + 4z^2$, replace x by 18, y by 4, and z by 2.

$$\begin{aligned} \frac{1}{2}x - 5y + 4z^2 &= \frac{1}{2}(18) - 5(4) + 4(2)^2 \\ &= 9 - 20 + 4(2 \times 2) \\ &= 9 - 20 + 16 \\ &= 25 - 20 \\ &= \_\_\_\_ \end{aligned}$$

If the domain of a variable is not specified, it is assumed to be R, the set of real numbers.

Self-correcting Exercise #1

Answers may be found on page 44 of this lesson.

1. Evaluate the variable expression $-5x^3$ when $x = -2$, 0, and 2.

   (a) When $x = -2$, $-5x^3 = -5(-2)^3 =$ ____________________

   (b) When $x = 0$, $-5x^3 =$ ____________________

   (c) When $x = 2$, $-5x^3 =$ ____________________

2. Evaluate the following expressions for the values of the variables that are specified.

   (a) $5x + 1$ when $x = -3$

   $5x + 1$

   $= 5(-3) + 1$

   $=$ _____ $+ 1$

   $=$ _____

   (b) $5(x + 1)$ when $x = -3$

   (c) $-2y^2 + 7$ when $y = 2$

   (d) $-4(m + n)^2$ when $m = -8$ and $n = 3$

   (e) $-2\sqrt{a + b - c}$ when $a = 9$, $b = -2$, and $c = 3$

   (f) $7x^2y - 5z^3$ when $x = 2$, $y = -1$, and $z = -2$

B. Terms of a Sum

Numbers which are added together to form a sum are called ADDENDS. These numbers can also be called the TERMS of a sum.

Variable expressions often represent sums. The terms in these sums always appear as individual numbers or variables or as the product or quotient of numbers and variables.

EXAMPLE: $-3a + \frac{b}{5} + c$

The three terms in this sum are $-3a$, $\frac{b}{5}$, and c.

Note that each term appears as an individual number or variable or as the product or quotient of numbers and variables.

In the above example, the first term appears as the product of the real number -3 and the variable ___. The second term is the quotient of the variable ___ and the real number ___. The third term is the variable ___.

Variable expressions involving the operation of subtraction can be thought of as sums so that individual terms can be identified.

EXAMPLE: $3a^2 - 6a - 5$

This variable expression could be written as the sum

$$3a^2 + (-6a) + (-5)$$

The terms of this sum are $3a^2$, $-6a$, and -5.

Name the terms in the variable expression $\frac{2a}{3} - 5b + c - 8d^2$.

________, $-5b$, ________, ________

Write a variable expression that has five terms.

____

Name these five terms.

________, ________, ________, ________, ________

Self-correcting Exercise #2

Answers may be found on page 44 of this lesson.

1. List the terms in each variable expression.

(a) $2b^2 - 3c + 8bc - 5$ $2b^2, -3c, 8bc, -5$

(b) $x + \frac{y}{z} - 3xz$ ____________

(c) $2(b - c) + 3d - 4(m + n)$ ____________

(d) $\frac{2(x + 3)}{7}$ ____________

(e) $a - \frac{3b}{4} + \frac{a + b}{2} + \frac{b^2 - a}{b}$ ____________

C. Factors of a Product

Numbers which are multiplied together to form a product are called FACTORS. Any product represents exactly one term.

Variable expressions often represent products. The factors in these products appear as individual numbers or variables or as the sum or difference of numbers and variables.

EXAMPLE: $7(x + 3)(y - 5)$

The three factors in this product are 7, $(x + 3)$, and $(y - 5)$.

Note that each factor appears as an individual number or variable or as the sum or difference of numbers and variables. The product $7(x + 3)(y - 5)$ represents one term.

In the above example, the first factor is the real number ___. The second factor is the sum of the variable ___ and the real number ___. The third factor is the difference of the variable ___ and the real number ___.

Name the factors in the variable expression $x(y + z)(y^2 - 3z)$.

__________, __________, __________

If a variable expression contains a squared factor, this means that the factor appears twice in the product. If it contains a cubed factor, this means that the factor is used three times in the product. For example,

$2x^2y$ means $2 \times x \times x \times y$ (Since 2 means 2' and y means y', 2 and y appear only once as factors.)

a^3b^2 means $a \times a \times a \times b \times b$

Fill in the blanks below.

$5(a - b)^2$ means $5 \times (\_\_\_) \times (\_\_\_)$

$6m^2n^2$ means $6 \times \_\_ \times \_\_ \times \_\_ \times \_\_$

$(x + y)^2(x - y)^2$ means $(\_\_\_\_)(\_\_\_\_)(\_\_\_\_)(\_\_\_\_)$

$2xy^2z^3$ means $2 \times \_\_\_ \times \_\_\_ \times \_\_\_ \times \_\_\_ \times \_\_\_ \times \_\_\_$

Variable expressions involving the operation of division can be thought of as products so that individual factors can be identified.

EXAMPLE: $\frac{3b^2}{c^3d}$

This variable expression could be written as the product

$$3b^2 \times \frac{1}{c^3d}$$

$$= 3 \times b \times b \times \frac{1}{c} \times \frac{1}{c} \times \frac{1}{c} \times \frac{1}{d}$$

In the example above, how many times does $\frac{1}{c}$ appear as a factor? __________ How many times does $\frac{1}{d}$ appear as a factor? __________

Name the factors of the variable expression $\frac{3mn}{r}$.

________, ________, ________, ________

Self-correcting Exercise #3

Answers may be found on page 44 of this lesson.

1. List the individual factors indicated in the following products.

   (a) $x^2(x - y)^2$ ____________________

   (b) $9a^2bc^2$ ____________________

   (c) $\frac{5xy}{z}$ ____________________

   (d) $\frac{3(m + n)}{m(m - n)}$ ____________________

   (e) $\frac{a^2}{3bc}$ ____________________

D. Numerical and Literal Coefficients

The NUMERICAL COEFFICIENT of an algebraic term is the number factor in the term. The LITERAL COEFFICIENT of an algebraic term contains all the variable factors in the term.

EXAMPLES:

In the term $15x^2y$, the numerical coefficient is 15 and the literal coefficient is x^2y.

In the term $\frac{3y^2}{2}$, the numerical coefficient is $\frac{3}{2}$ and the literal coefficient is y^2.

IMPORTANT! In the term $-ab$, the numerical coefficient is -1 and the literal coefficient is ab.

In the term y^2, the numerical coefficient is 1 and the literal coefficient is y^2.

Algebraic terms are called LIKE TERMS if they have the same literal coefficient.

EXAMPLES:

$-5abc$, $\frac{2}{3}abc$, $\frac{3}{4}abc$ are like terms because they all have the literal coefficient abc.

$-3xy$ and $4x$ are <u>not</u> like terms since they have different literal coefficients. $-3xy$ has a literal coefficient of xy and $4x$ has a literal coefficient of x.

Self-correcting Exercise #4

Answers to this exercise may be found on page 45 of this lesson.

1. State the numerical coefficient and the literal coefficient of each of the following terms.

	Numerical Coefficient	Literal Coefficient
(a) $-7x^2yz^2$	__________	__________
(b) $\frac{-3a}{5}$	__________	__________
(c) $5 \times 7 \times a$	__________	__________
(d) $\frac{-x}{2}$	__________	__________
(e) $\frac{3 \times 4 \times y^2}{5}$	__________	__________
(f) a^2bc^3	__________	__________

2. Write "yes" in the blank if the pair of terms are like terms and "no" if they aren't.

(a) $\frac{5y}{2}$, $8y$ __________ (b) $3(a + b)$, $2b$ __________

(c) $6x^2y$, $4xy^2$ __________ (d) mn, $\frac{4mn}{7}$ __________

(e) $5a^2$, $5a$ __________ (f) $3(x - y)$, $(x - y)$ __________

EXERCISE - Variable Expressions

1. Fill in the blanks.

(a) A set of values which may be substituted for a variable is called the ______________________ of the variable.

(b) When $y = -2$, the expression $\frac{1}{2}y - 5$ has a numerical value of __________.

(c) The expression $\frac{2}{3}(a - 3)$ instructs us to ______________ 3 from a number and then ____________________ the result by ________.

(d) In the term $\frac{-3x^3y^4}{z^2}$, x appears as a factor _____________ times.

(e) A ____________________ is a symbol which may represent any of the elements of a specified set.

(f) In the term $-3(x + 2)(x - 7)^2$, the factors are _____, _____, _____, and _____.

(g) The variable terms $-3a^2b$ and $\frac{4}{5}a^2b$ are __________________ terms because they have the same __________________________ coefficient.

(h) In the term $-5x^2y^3$, the ______________________________ coefficient is -5 and the _________________________ coefficient is x^2y^3.

(i) In the product $-2ab$; -2, a, and b are called the _________________ of the product.

(j) In the sum $c + b + 4d$; c, b, and 4d are called the ____________ of the sum.

(k) The algebraic term $-xyz^3$ has a numerical coefficient of __________.

2. Put a check mark beside each expression that represents one term.

(a) $\frac{a - 2b}{5}$ ✓ (b) $3abc - d$ _____ (c) $-3xy^2z$ _____

(d) $(a+b)(c+d)$ _____ (e) $1 - x$ _____ (f) $\frac{3mn}{s}$ _____

(g) $2(a + b - c)$ _____ (h) $\frac{x}{3} + \frac{y}{2} - \frac{z}{5}$ _____ (i) $\frac{a^2 + 2ab}{3b}$ _____

3. Evaluate each of the following expressions for all the values from the domain of the variable.

(a) Evaluate $3t^2 - t - 4$ when $t \in \{-1, 0, 2\}$.

(i) When $t = -1$, $3t^2 - t - 4 = $ $3(-1)^2-(-1)-4=3+1-4=4-4=0$

(ii) When $t = 0$, $3t^2 - t - 4 = $ ______

(iii) When $t = 2$, $3t^2 - t - 4 = $ ______

(b) Evaluate $x^3 + 3x$ when $x \in \{-2, -1, 3\}$.

(i) When $x = -2$, $x^3 + 3x = $ ______

(ii) ______

(iii) ______

(c) Evaluate $\frac{t + 6}{t - 5}$ when $t \in \{-6, -3, 0, 5\}$.

(i) When $t = -6$, $\frac{t + 6}{t - 5} = $ ______

(ii) ______

(iii) ______

(iv) ______

4. Write each expression as the product of factors of the first degree.

(a) $\frac{3x^3}{5y^2} = $ $\frac{3}{5} \times x \times x \times x \times \frac{1}{y} \times \frac{1}{y}$

(b) $-8\,(a + b)^2c^3 = $ ______

(c) $\frac{(x - y)^2z^2}{y^2} = $ ______

(d) $4mnt^2 = $ ______

(e) $\frac{(x + y)^3 z^3}{3y} = $ ______

5. Evaluate each expression for the values that are specified for the variables.

(a) $z - (x - y)$ when $x = 5$, $y = -2$, and $z = 8$.

$z - (x - y)$

$= 8 - [5 - (-2)]$

$= 8 - [\_\_ + \_\_]$

$= 8 - \_\_$

$= \_\_$

(b) $(3xz)^3$ when $x = -1$ and $z = 2$.

$(3xz)^3$

$= (\_\_ \times \_\_ \times \_\_)^3$

$= (\_\_)^3$

$= \_\_ \times \_\_ \times \_\_$

$= \_\_$

(c) $\frac{x}{y} + z$ when $x = -4$, $y = 8$ and $z = \frac{5}{2}$.

$\frac{x}{y} + z$

$=$

(d) $5a^2b + 3bc$ when $a = 2$, $b = -3$, and $c = 4$.

$5a^2b + 3bc$

$= 5(\_\_)^2(\_\_) + 3(-3)(\_\_)$

$=$

(e) $-3(x + y)^2$ when $x = 5$ and $y = -7$.

$-3(x + y)^2$

$= -3(\_\_ + \_\_)^2$

$=$

(f) $ab - bc + 5ac$ when $a = -2$, $b = 3$, and $c = 4$.

$ab - bc + 5ac$

$= (-2)(3) - (\_\_)(4) + 5(\_\_)(\_\_)$

$=$

Topic Two: Operating With Variable Expressions

A. Adding Variable Expressions

Variable terms can be added only if they are like terms. That is, they must have the same literal coefficient.

The distributive property applied in reverse allows us to combine like terms by adding and arrive at a single term which represents the sum. The numerical coefficient of the sum is equal to the sum of the numerical coefficients of the like terms. The literal coefficient of the sum is the same as the literal coefficient of the like terms.

EXAMPLE: Write the sum 9xy + 3xy + 2xy + xy as a single term.

Solution

$$9xy + 3xy + 2xy + xy$$

Note that this sum contains four like terms (since each has a literal coefficient of xy). The distributive property can be applied in reverse so that we can add the numerical coefficients of the 4 terms.

$= (9 + 3 + 2 + 1)xy \longleftarrow$ Think this step. Don't write it.

$= \underline{\underline{15xy}}$

Fill in the blanks in the chart below to find the following sums.

Expression	Think Step	Sum
$3a + 5a$	(3+5)a	8a
$2x + 4x + 11x$	__________	____
$7y + y + 9y$	__________	____
$2x^2y + 4x^2y + 9x^2y$	__________	____
$2ab + ab$	__________	____
$a^2 + a^2 + a^2$	__________	____
$\frac{1}{2}b + \frac{1}{2}b$	__________	____
$3.5a^2b^2 + 0.5a^2b^2$	__________	____

The commutative and associative properties of addition allow us to change the order and manner of grouping terms in a sum so that like terms can be grouped together. Once like terms are placed beside each other, the distributive property can be used to add their numerical coefficients.

EXAMPLE: Add like terms in the sum $3a + \frac{1}{2}b + \frac{2}{3}a + 4c + 2b + 5c$.

<u>Solution</u>

$$3a + \frac{1}{2}b + \frac{2}{3}a + 4c + 2b + 5c$$

$$= (3a + \frac{2}{3}a) + (\frac{1}{2}b + 2b) + (4c + 5c)$$ $\left\{\text{Commutative and Associative Properties}\right.$

$$= (3 + \frac{2}{3})a + (\frac{1}{2} + 2)b + (4 + 5)c \longleftarrow \text{Distributive Property (in reverse)}$$

$$= \frac{11}{3}a + \frac{5}{2}b + 9c$$

Now, simplify each of the sums below by combining like terms.

1. $3a^2 + 2a + a^2 + 5a + 4 + 7a^2$

$= (3a^2 + a^2 + \_a^2) + (2a + \_a) + \_\_$

$= \_\_a^2 + \_\_a + \_\_$

2. $4x + 3y + 5x + 9y$

$= (\_x + \_\_x) + (\_y + \_\_y)$

$= \_\_x + \_\_y$

3. $-2ab + 3b^2 + 2a + 5ab + 4b^2 + 5a$

$= (-2ab + \_\_) + (3b^2 + \_\_) + (2a + \_\_)$

$= \_\_ab + \_\_b^2 + \_\_a$

4. $2x^3 + 5x^2 + 3x + 7x + 9x^3$

$= (2x^3 + \_\_) + 5x^2 + (3x + \_\_)$

$= \_\_\_ + \_\_\_ + \_\_\_$

B. Subtracting Variable Expressions

Remember that multipliplication distributes over subtraction in the set of real numbers. Thus, the distributive property applied in reverse also allows us to subtract like terms and arrive at a single term which represents the difference. The numerical coefficient of the difference is equal to the difference of the numerical coefficients of the like terms. The literal coefficient of the difference is the same as the literal coefficient of the like terms.

EXAMPLE: Write the difference $3a^2b - 9a^2b$ as a single term.

Solution

$3a^2b - 9a^2b$

Apply the distributive property of multiplication over subtraction in reverse.

$= (3 - 9)a^2b$ ←—— Think this step. Don't write it.

$= \underline{\underline{-6a^2b}}$

Fill in the blanks in the chart below in order to find the following differences.

Expression	Think Step	Difference
$-2xy - 5xy$	$(-2 - 5)xy$	$-7xy$
$11a^2 - a^2$	________	______
$6mn - 10mn$	________	______
$6x^2y - 7x^2y$	________	______
$-3r - 5r$	________	______

Differences of terms can be thought of as sums so that the commutative and associative properties of addition can be used to group like terms. (Recall that subtraction is neither commutative nor associative.)

EXAMPLE: Combine like terms in the expression $8a - 3b + 5a - 2b$.

Solution

$8a - 3b + 5a - 2b$

Think of each difference as being a sum.

$= 8a + (-3b) + 5a + (-2b)$ ←——Think this step. Don't write it.

Now the commutative and associative properties of addition can be used to group like terms.

$= (8a + 5a) + (-3b - 2b)$

$= 13a + (-5b)$

$= \underline{\underline{13a - 5b}}$

Now, simplify the following expressions by combining like terms.

1. $3a^2 - 12ab + 7ab - 4a^2 + 2b - 6a^2 - 9b + 3ab$

$= (3a^2 - \_\_a^2 - \_\_a^2) + (-12ab + \_\_ab + \_\_ab) + (2b - \_\_b)$

At this point, count the terms to make sure that you haven't missed any.

$= \_\_a^2 - \_\_ab - \_\_b$

2. $x^2 + 4x - 9 - 3x^2 - 9x + 3 + 4x^2 + 8x - 7$

$= (\_\_ - \_\_ + 4x^2) + (\_\_\_\_\_\_\_\_ + 8x) + (\_\_\_\_\_\_\_\_ - 7)$

$=$ ________________

3. $-3m^2 - n^2 + 3m^2 + 4mn - 5n^2 - 8mn + 3n^2 - 5mn$

$= (\_\_m^2 + \_\_m^2) + (-n^2 - \_\_n^2 + \_\_) + (\_\_\_\_ - 8mn - \_\_\_\_)$

$=$ ________________

If you are asked to subtract two variable expressions, each containing a number of terms, you must first know how to find the additive inverse of the variable expression you are subtracting.

In our discussion of the set of real numbers, we found that every real number has an opposite which is called its ADDITIVE INVERSE. One real number is the additive inverse of another if their sum is zero. For example, $3\sqrt{2}$ and $-3\sqrt{2}$ are additive inverses of each other since $(3\sqrt{2}) + (-3\sqrt{2}) = 0$.

Similarly, every variable expression has an additive inverse. The sum of a variable expression and its additive inverse is zero.

EXAMPLE: Find the additive inverse of $(a + b - c)$.

Solution

We must find an expression which when added to $(a + b - c)$ will equal zero.

$$(a + b - c) + (\boxed{}) = 0$$

The above condition is satisfied if the unknown expression is $(-a - b + c)$.

i.e.
$$\begin{aligned} &(a + b - c) + (-a - b + c) \\ &= (a - a) + (b - b) + (-c + c) \\ &= 0a + 0b + 0c \\ &= 0 \end{aligned}$$

Therefore, the additive inverse of $(a + b - c)$ is $(-a - b + c)$.

In the example at the bottom of page 16, note that the additive inverse of the expression $a + b - c$ could have been determined by taking the additive inverse of each term. The additive inverse of a is $-a$. The additive inverse of b is $-b$. The additive inverse of $-c$ is c. Therefore, the additive inverse of $a + b - c$ is $-a - b + c$.

In general, the additive inverse of any variable expression can be determined by taking the additive inverse of each of its terms.

EXAMPLES:

1. The additive inverse of $-x^2 + 7x - 9$ is $x^2 - 7x + 9$.
   <u>Check</u>
   $$\begin{aligned} & (-x^2 + 7x - 9) + (x^2 - 7x + 9) \\ = & (-x^2 + x^2) + (7x - 7x) + (-9 + 9) \\ = & \ 0 \end{aligned}$$

2. The additive inverse of $\frac{-3}{2}x + \frac{-1}{2}y$ is $\frac{3}{2}x + \frac{1}{2}y$
   <u>Check</u>
   $$\begin{aligned} & \left(\frac{-3}{2}x + \frac{-1}{2}y\right) + \left(\frac{3}{2}x + \frac{1}{2}y\right) \\ = & \left(\frac{-3}{2}x + \frac{3}{2}x\right) + \left(\frac{-1}{2}y + \frac{1}{2}y\right) \\ = & \ 0 \end{aligned}$$

Similarly,

1. The additive inverse of $-a + b - c$ is a - ___ + ___.

2. The additive inverse of $x - y + z$ is $-x$ + ___ - ___.

3. The additive inverse of $x^2 - xy + y^2 - x^2y^2$ is ___ + ___ - ___ + ___.

Now that we know how to find the additive inverse of a variable expression, we can subtract two variable expressions that contain more than one term.

Recall that in order to subtract one real number from another, we must add the additive inverse of the subtrahend to the minuend. For example, the subtraction question $4 - 7$ can be changed to the addition question $4 + (-7)$ and the rules for adding real numbers can then be applied.

i.e. $4 - 7 = 4 + (-7) = -3$

(Operation is changed to addition) (Additive inverse of 7 is -7.)

Subtraction of variable expressions can be defined in a similar manner. In order to subtract one variable expression from another, we must add the additive inverse of the second expression to the first. Once the subtraction question has been changed to an addition question, the rules for adding variable terms can then be applied.

EXAMPLE: Subtract $-5a + 2b - 3c$ from $-8a + 3b - 6c$.

<u>Solution</u>

$(-8a + 3b - 6c) - (-5a + 2b - 3c)$

Change the operation to addition and give the additive inverse of the second expression. (The additive inverse of $-5a + 2b - 3c$ is $5a - 2b + 3c$.)

$= (-8a + 3b - 6c) + \underbrace{(5a - 2b + 3c)}$

(Operation is changed to addition) (This is the additive inverse of $-5a+2b-3c$.)

Now, add as before by grouping like terms.

$= (-8a + 5a) + (3b - 2b) + (-6c + 3c)$

$= \underline{\underline{-3a + b - 3c}}$

<u>Self-correcting Exercise #5</u>

Answers may be found on page 45 of this lesson.

1. Give the additive inverse of each of the following expressions.

(a) x ________________

(b) $a + b$ ________________

(c) $-y$ ________________

(d) $-a + b - c^2$ ________________

(e) $a^2 - 2ab + b^2$ ________________

(f) $-a^2 - b^2 + bc + 1$ ________________

2. Perform each subtraction by changing the operation to addition and giving the additive inverse of the second expression. Then collect like terms.

(a) $(5a - 6) - (3a - 2)$

$= (5a - 6) + (\_\_ + 2)$

$= (5a - 3a) + (-6 + \_\_)$

$= \_\_ - \_\_$

(b) $(5x + 2y + 3z) - (-x + 2y - 5z)$

$= (5x + 2y + 3z) + (\_\_ - \_\_ + \_\_)$

$=$

(c) $(3x^2 - 2y^2 - 4xy) - (7y^2 - xy)$

(d) $(2x - y + 3) - (-2y - 5)$

3. Simplify the following expressions by combining like terms.

(a) $6b^2 + 2b^2 - 9b^2$

$= \_\_\_\_$

(b) $\sqrt{2}a + 5\sqrt{2}a$

$= \_\_\_\_$

(c) $3a^2 + 2a - a^2 + 5a - 4a^2$

$=$

(d) $2s + 3r - s - r$

$=$

(e) $5a^4 - 9a^3 - 12a^2 - 5a^3 + 5a^2 - 6a^4 - 3a^4 + 2a^3 - 6a^2$

$=$

(f) $(2x - y + 3) - (-2y + 5) - (3x - 4y)$

$=$

(g) $(3a + b - 5c) + (2a - 5b + 4c) - (3a - 2b - 5c)$

$=$

C. Multiplying Variable Expressions

When multiplying variable terms, you must often use the Product of Powers Property. Recall that this property tells you how to find the product of two powers with like bases. This can be done by retaining the common base and <u>adding</u> the exponents.

Product of Powers Property
$a^m . a^n = a^{m+n}$

For example,

Add the exponents.

$x^2 \times x^3 = x^{2+3} = x^5$

Retain the common base.

$y^9 \times y^{-4} = y^{9+(-4)} = y^5$

$a^3 \times a = a^{3+1} = a^4$

Find the following products.

1. $z^5 \times z^2 = z^{\_\_+\_\_} = z^{\_\_}$

2. $m^8 \times m^{-6} = m^{\_\_+\_\_} = m^{\_\_}$

3. $c^{-4} \times c^{-3} = c^{-4+\_\_} = c^{\_\_}$

4. $x^2 \times x^2 = x^{\_\_+\_\_} = x^{\_\_}$

5. $y^4 \times y^5 \times y = y^{\_\_+\_\_+\_\_} = y^{\_\_}$

Sometimes you must find the product of variable terms that have numerical coefficients other than 1. In such cases, you must use the commutative and associative properties of multiplication to group numerical factors together and group literal factors with like bases together. The numerical coefficient of the product can be determined by finding the product of all the numerical factors. The literal coefficient of the product can be written in simplest form by applying the Product of Powers Property to variable factors with like bases.

EXAMPLE: Write the product $(5x^2y^3)(-2x^5y^4)$ in simplest form.

Solution

$(5x^2y^3)(-2x^5y^4)$

Group the numerical factors 5 and -2 together.
Group the factors x^2 and x^5 (that have base x) together.
Group the factors y^3 and y^4 (that have base y) together.

$$= \underbrace{(5 \times -2)}_{\text{numerical factors}}\underbrace{(x^2 \times x^5)}_{\text{factors with base } x}\underbrace{(y^3 \times y^4)}_{\text{factors with base } y}$$

Now, multiply the numerical factors and use the Product of Powers Property to multiply the variable factors with like bases.

$$= (-10)(x^{2+5})(y^{3+4})$$

$$= \underline{\underline{-10x^7y^7}}$$

Find the following products.

1. $-2b \times 3c = (-2 \times 3)bc =$ _____bc

2. $-6x \times -4x = (-6 \times$ ___$)(x \times x) =$ ___x^{1+1} $=$ ____x^{-}

3. $3a^2b^2 \times -8ab^3 = (3 \times$ ___$)(a^2 \times a)(b^2 \times b^3) =$ ___a^{2+1} b^{2+3} $=$ ____$a^{-}b^{-}$

4. $6y^3 \times 5y^{-8} = ($___$\times 5)(y^3 \times$ ___$) =$ ___y^{3+-} $=$ ____y^{-}

5. $-9x^3y^2z^5 \times \frac{1}{3}xy^2z^{-3} = (— \times \frac{1}{3})(x^3 \times$ ___$)(y^2 \times$ ___$)(z^5 \times$ ___$) =$ ___$x^{-}y^{-}z^{-}$

The distributive property may be used to multiply variable expressions containing more than one term.

EXAMPLE 1: Find the product of $-7y^2$ and $-3y^3 + 7y - 1$.

Solution

The multiplier $-7y^2$ must be distributed over the three terms ($-3y^3$, $7y$, and -1) in the variable expression $-3y^3 + 7y - 1$.

$(-7y^2)(-3y^3 + 7y - 1)$

$= (-7y^2)(-3y^3) + (-7y^2)(7y) + (-7y^2)(-1) \longleftarrow$ Distributive Property

Simplify each term by finding the products of the expressions involved.

$= (-7 \times -3 \times y^{2+3}) + (-7 \times 7 \times y^{2+1}) + (-7 \times -1 \times y^2)$

$= \underline{\underline{21y^5 - 49y^3 + 7y^2}}$

Find each of the following products by multiplying the first factor by <u>every</u> term in the second factor.

1. $4x^2(5x^3 + 3x^2 - 5x) = (4x^2)(5x^3) + (4x^2)(\_\_) + (4x^2)(-5x)$

$$= 20x^{\_\_} + \_\_x^{\_\_} - 20x^{\_\_}$$

2. $ab(a^2 - b^2 + 5b - 9ab) = (ab)(a^2) + (ab)(-b^2) + (\_\_)(5b) + (\_\_)(-9ab)$

$$= a^{\_\_}b - ab^{\_\_} + 5ab^{\_\_} - 9a^{\_\_}b^{\_\_}$$

3. $-3x(x^2 - 2x + 1) = (-3x)(x^2) + (\_\_)(-2x) + (\_\_\_)(1)$

$$= -3x^{\_\_} + 6x^{\_\_} - \_\_\_\_$$

EXAMPLE 2: Find the product of $(2x + 3)$ and $(5x - 2)$.
<u>Solution</u>

$$(2x + 3)(5x - 2)$$

The multiplier $(2x + 3)$ must be distributed over the two terms ($5x$ and -2) in the second factor $(5x - 2)$

$$= (2x + 3)(5x) + (2x + 3)(-2)$$

Use the distributive property again to find the products $(2x + 3)(5x)$ and $(2x + 3)(-2)$.

$$= (2x)(5x) + (3)(5x) + (2x)(-2) + (3)(-2)$$

$$= 10x^2 \quad + 15x \quad - 4x \quad - 6$$

Note that the two middle terms are like terms and can be combined.

$$= \underline{\underline{10x^2 + 11x - 6}}$$

In the example above, four separate products had to be found. These products are obtained by pairing each term in the first factor with each term in the second factor. These products can be written down immediately by using the following procedure. (Arrows show the order in which the terms can be multiplied.)

$$(2x + 3)(5x - 2) = \overset{1}{(2x)(5x)} + \overset{2}{3(5x)} + \overset{3}{2x(-2)} + \overset{4}{3(-2)}$$

$$= 10x^2 \quad + 15x \quad - 4x \quad - 6$$

$$= \underline{\underline{10x^2 + 11x - 6}}$$

Use this short-cut method to find the following products.
(Combine like terms if there are any.)

1. $(3a + 4b)(5a + b) = (3a)(5a) + (4b)(5a) + (3a)(b) + (4b)(b)$

$= \_\_\_a^2 + \_\_\_ab + \_\_\_ab + \_\_\_b^2$

$= \_\_\_a^2 + \_\_\_ab + \_\_\_b^2$

2. $(2x - 3y)(2x + 3y) = (2x)(\_\_) + (-3y)(\_\_) + (\_\_)(3y) + (-3y)(\_\_)$

$= \_\_\_x^2 - \_\_\_xy + \_\_\_xy - \_\_\_y^2$

$= \_\_\_x^2 - \_\_\_y^2$

3. $(2m - n)(3r + s) = (\_\_)(3r) + (-n)(\_\_) + (2m)(\_\_) + (-n)(\_\_)$

$= \_\_\_mr - \_\_\_nr + \_\_\_ms - \_\_\_$

Self-correcting Exercise #6

Answers may be found on page 46 of this lesson.

1. Fill in the blanks in the chart in order to find each product.

	Product of Numerical Factors	Product of x Factors	Product of y Factors	Entire Product
$(5x^2y)(-10x^7y^3)$	$5 \times -10 = -50$	$x^2 \times x^7 = x^9$	$y \times y^3 = y^4$	$-50x^9y^4$
$(-2x^3y^3)(7x^{-5}y^2)$	___	___	___	___
$\left(\frac{1}{3}xy\right)(-6x^4y^{-3})$	___	___	___	___
$(-xy)(5xy)$	___	___	___	___
$(6x^2y)(-3xy^3)$	___	___	___	___

2. Find the following products.

(a) $x^5 \times x^4$ = ______

(b) $(-3a^3)(7a^5)$ = ______

(c) $(3b)(-5b^4x)$ = ______

(d) $\frac{1}{2}xy \quad (-2xy^3)$ = ______

(e) $(-4m^2n^3)(mn^4)(-2m^3n)$ = ______

(f) $(6cd)(-5c^2)(3d^2)$ = ______

(g) $(-5a^2b^3c)(7ac^5)$ = ______

(h) $(-x^2y^2z^3)(3xy^3)$ = ______

(i) $2a(a^2 - ab + b^2)$

$= (2a)(a^2) + (2a)(\_\_\_) + (2a)(\_\_\_)$

$= \_\_\_ - \_\_\_ + \_\_\_$

(j) $-3x(y^2 + y - 7)$

=

(k) $(2a + b)(3a^2 + 5)$

$= (2a)(3a^2) + (\_\_\_)(\_\_\_) + (\_\_\_)(\_\_\_) + (\_\_\_)(\_\_\_)$

$= \_\_\_ + \_\_\_ + \_\_\_ + \_\_\_$

(l) $(2x + 5)(3x - 2)$

$= (2x)(3x) + (\_\_\_)(\_\_\_) + (\_\_\_)(\_\_\_) + (\_\_\_)(\_\_\_)$

=

(m) $(a^2 - 2ab)(a^2 + 2ab)$

=

D. <u>Dividing Variable Expressions</u>

When dividing variable terms, you must use the Quotient of Powers Property. Recall that this property tells you how to find the quotient of two powers with like bases. This can be done by retaining the common base and <u>subtracting</u> the exponents.

Quotient of Powers Property

$$\frac{a^m}{a^n} = a^{m-n}$$

For example,

Subtract the exponents.

$$\frac{y^3}{y^2} = y^{3-2} = y^1 = y$$

Retain the common base.

If the exponent in the denominator is larger than the exponent in the numerator we often apply an alternate form of the Quotient of Powers Property so that the result will have a positive exponent. This alternate form tells us that

$$\boxed{\frac{a^m}{a^n} = \frac{1}{a^{n-m}}}$$

For example,

$$\frac{y^3}{y^8} = \frac{1}{y^{8-3}} = \frac{1}{y^5}$$

Find the following quotients. (Express answers with positive exponents.)

1. $\frac{z^2}{z^5} = \frac{1}{z^{\_-\_}} = \frac{1}{z^{\_\_}}$

2. $\frac{a^8}{a^4} = a^{\_-\_} = a^{\_\_}$

3. $\frac{x^4}{x^4} = x^{\_-\_} = x^{\_\_} =$ ____

4. $\frac{m^3}{m^{-3}} = m^{\_-\_} = m^{\_\_}$

5. $\frac{y^4}{y^3} =$

When you are finding the quotient of variable terms that have numerical coefficients other than 1, you can write the quotient as a product so that the commutative and associative properties of multiplication can be applied. These properties allow you to group numerical factors together and variable factors with like bases together. The literal coefficient of the quotient can be written in simplest form by applying the Quotient of Powers Property to variable factors with like bases.

EXAMPLE: Write the quotient $\frac{-8x^3y^3}{6xy^4}$ in simplest form.

Use positive exponents only in the answer.

Solution

$$\frac{-8x^3y^3}{6xy^4}$$

Write the quotient as a product.

$$= -8 \times x^3 \times y^3 \times \frac{1}{6} \times \frac{1}{x} \times \frac{1}{y^4}$$

Group the numerical factors -8 and $\frac{1}{6}$ together.

Group the factors x^3 and $\frac{1}{x}$ together.

Group the factors y^3 and $\frac{1}{y^4}$ together.

$$= \underbrace{\left(-8 \times \frac{1}{6}\right)}_{\text{numerical factors}} \underbrace{\left(x^3 \times \frac{1}{x}\right)}_{\text{factors with base } x} \underbrace{\left(y^3 \times \frac{1}{y^4}\right)}_{\text{factors with base } y}$$

$$= \frac{-8}{6} \times \frac{x^3}{x} \times \frac{y^3}{y^4}$$

Divide the numerical factors and use the Quotients of Powers Property to divide variable factors with like bases.

$$= \frac{-4}{3} \times x^{3-1} \times \frac{1}{y^{4-3}}$$

Divide in this manner so exponent of y will be positive.

$$= \boxed{\frac{-4x^2}{3y}}$$

When doing questions of this type, you should begin at this step where numerical factors are divided and variable factors with like bases are divided. Just keep in mind that the commutative and associative properties of multiplication allow you to do this.

Find the following quotients. Divide the variable factors in such a way that the exponents will be positive.

1. $\frac{15x^3}{3x^2} = \frac{15}{3} \times \frac{x^3}{x^2} = 5 \times x^{3-\underline{\quad}} = 5\underline{\quad}$

2. $\frac{4y^4}{16y^9} = \frac{4}{16} \times \frac{y^4}{y^{\underline{\quad}}} = \frac{1}{4} \times \frac{1}{y^{\underline{\quad}-\underline{\quad}}} = \frac{1}{4y^{\underline{\quad}}}$

3. $\frac{9z^5}{3z} = \frac{9}{\underline{\quad}} \times \frac{z^{\underline{\quad}}}{z} = \underline{\quad} \times z^{\underline{\quad}-\underline{\quad}} = \underline{\quad}z^{\underline{\quad}}$

4. $\frac{-18r^7s^3t^5}{16r^3s^5t^3} = \frac{-18}{\underline{\quad}} \times \frac{r^7}{r^{\underline{\quad}}} \times \frac{s^{\underline{\quad}}}{s^{\underline{\quad}}} \times \frac{t^{\underline{\quad}}}{t^{\underline{\quad}}} =$

Self-correcting Exercise #7

Answers may be found on page 48 of this lesson.

1. Fill in the blanks in the chart in order to find each quotient. (Express answers with positive exponents.)

	Quotient of Numerical Factors	Quotient of x Factors	Quotient of y Factors	Entire Quotient
$\frac{27xy^9}{-3x^4y^4}$	$\frac{27}{-3} = -9$	$\frac{x}{x^4} = \frac{1}{x^3}$	$\frac{y^9}{y^4} = y^5$	$\frac{-9y^5}{x^3}$
$\frac{36x^4y^5}{9x^5y^2}$	______	______	______	______
$\frac{3x^4y^3}{\frac{1}{2}x^2y}$	______	______	______	______
$\frac{-48x^6y^5}{-16x^9y^4}$	______	______	______	______
$\frac{5xy}{25x^3y^4}$	______	______	______	______

2. Find the following quotients. Express answers with positive exponents.

(a) $\frac{x^4}{x^2} = x^{\_\_}$ (b) $\frac{m}{m^5} = \frac{1}{m^{\_\_}}$

(c) $\frac{15y^3}{-5y^2} =$ (d) $\frac{-2a}{4a^2} =$

(e) $\frac{-12a^2b^2c^4}{-8b^2c^5}$

$= \frac{-12}{-8} \times a^2 \times \frac{b^2}{b^2} \times \frac{c^4}{c^5}$

$=$

(f) $\frac{-20m^2}{45m^3n}$

$=$

EXERCISE - Operating With Variable Expressions

1. Fill in the blanks.

(a) The __________ property allows us to write the sum $3abc + 5abc$ as the single term $8abc$.

(b) The __________ and __________ properties of addition allow us to group like terms in the sum $a + b + 2a + 3b$.

(c) The __________ of __________ property allows us to represent the quotient $\frac{x^7}{x^4}$ by the single power x^3.

(d) In order to subtract $(3x^2 - 5x + 7)$ from $(9x^2 + 3x - 8)$, we must __________ the __________ inverse of $(3x^2 - 5x + 7)$ to $(9x^2 + 3x - 8)$.

(e) The sum of a^3 and a^3 is __________.

The difference of a^3 and a^3 is __________.

The product of a^3 and a^3 is __________.

The quotient of a^3 and a^3 is __________.

(f) When adding variable expressions, we group __________ terms and then use the __________ property to add the __________ coefficients of these terms.

(g) The additive inverse of a variable expression is determined by taking the ____________ ____________ of each term. The additive inverse of $x - y - 2z$ is ______________________.

(h) Expressions which are added together to form a sum are called addends or ____________.

(i) Expressions which are multiplied together to form a product are called ____________.

2. Simplify by combining like terms.

(a) $4y + 9y$ = 13y	(b) $-6x + 5x$ = ____
(c) $3ab - 5ab$ = ____	(d) $-2m - 8m$ = ____
(e) $7abc - 7abc$ = ____	(f) $-3r + 8r$ = ____
(g) $-3x + 5x + 12x$ = ____	(h) $5m + 2n - 4n - m$ = ____

(i) $-9x + 4y - 3z - 5y + 12x + 2y$

$= (-9x + 12x) + (4y - 5y + 2y) - 3z$

$=$

(j) $3x^2y - 12x^2y + 5x^2y + 8x^2$

$=$

(k) $4ab - 5b^2 + 2b^2 - 3ab^2 + 7b^2 - 8ab$

$=$

(l) $x^2 + 6xy - 5y^2 + 3x^2 - 9xy + 6y^2 - 2x^2 - y^2$

$=$

(m) $(r^2 + r + 6) - (2r^2 - 2r - 5)$

$= (r^2 + r + 6) + (-2r^2 + \_\_ + \_\_)$ *Always include this step in order to avoid making sign errors.*

$= (r^2 - 2r^2) + (r + \_\_\_) + (6 + \_\_\_)$

$=$ ________________________

(n) $(2x^2 + 3x - 5) + (x^2 - 5x + 7) - (3x^2 + 2x + 1)$

$= (2x^2 + 3x - 5) + (\_\_\_\_\_\_\_\_\_\_) + (\_\_\_\_\_\_\_\_\_\_)$ *Additive inverse of $(3x^2 + 2x + 1)$*

$=$

(o) $(a - 2b) - (a + b) + (2a - 5b) - (-a - b)$

$=$

3. Multiply.

(a) $y^{11} \times y^6 = y^{17}$

(b) $-x^2 \times x^3 = \_\_\_\_$

(c) $2x^2 \times 3xy = \_\_\_\_$

(d) $-4ab \times -2a = \_\_\_\_$

(e) $y^3 \times y^{-4} = \_\_\_\_$

(f) $\left(\frac{1}{2}ab\right)(8b^3) = \_\_\_\_$

(g) $(-5xy^2)(x^{-3}y^{-4}) = \_\_\_\_$

(h) $3ab^2c \times 2a^2bc^2 = \_\_\_\_$

(i) $(-2x^2y)(3xy^2)(7x^2y^2)$

$=$

(j) $\frac{1}{2}y^3 \times \frac{-3}{4}z^3 \times \frac{2}{3}yz^2$

$=$

(k) $(4mn)(-n^2)(m^2n)$

$=$

(l) $-2a \times 3ab \times \frac{-1}{2}b^2$

$=$

(m) $4x(x + y)$

$= 4x(x) + 4x(y)$

$= 4x^2 + 4xy$

(n) $3b(a - b)$

$=$

(o) $2m(-m^2 + 3mn - n^2)$

$= 2m(-m^2) + 2m(3mn) + 2m(-n^2)$
$= -2m^3 + 6m^2n - 2mn^2$

(p) $-4xy(2x^3 - 3x^2y^2)$

$=$

(q) $-2(a + b - c + d)$

$=$

(r) $6abc(a^2 - 2bc)$

$=$

(s) $-1(x - y + z)$

$=$

(t) $-3y(y - 2z)$

$=$

(u) $(a - 5)(a + 3)$

$= (a)(a) + (-5)(a) + (a)(3) + (-5)(3)$
$= a^2 - 5a + 3a - 15$
$= a^2 - 2a - 15$

(v) $(3x - 7)(4x + 1)$

$=$

(w) $(3a + b)(2a - 5b)$

$=$

(x) $(xy + 5z)(xy - 5z)$

$=$

(y) $(-y + z)(y - z)$

$=$

(z) $(2x - 3)(3x + 4)$

$=$

4. Divide. (Write answers with positive exponents.)

(a) $\frac{y^3}{y^2}$ = ______

(b) $\frac{x^2}{x^4}$ = ______

(c) $\frac{15m^5}{-3m}$ = ______

(d) $\frac{-2y^4}{8y}$ = ______

(e) $\frac{12x^3y^3}{24xy} = \frac{12}{24} \times \frac{x^3}{x} \times \frac{y^3}{y} =$

(f) $\frac{-4ab}{2b}$ =

(g) $\frac{36m^2}{12m^3}$ =

(h) $\frac{-20ab^2c}{4ab^2}$ =

(i) $\frac{17f^7g^2}{-51fg^9}$ =

(j) $\frac{-3m^2n}{15m^5n^6}$ =

(k) $\frac{16x^4y^2}{12x^3y^8}$ =

(l) $\frac{-49x^4y^3z^4}{-7x^2yz^3}$ =

(m) $\frac{-16a^{100}}{4a^{99}}$ =

Topic Three: Powers of Variable Terms

A. Integral Powers

Any power that has a base which is a real number and an exponent that is a positive integer is called a POSITIVE INTEGRAL POWER. Give examples of four positive integral powers.

$\left(\frac{-1}{2}\right)^5$________, __________, __________, __________

The exponent of a positive integral power tells us how many times the base is to be used as a factor. For example,

5^3 means $5 \times 5 \times 5$.

$(-2)^5$ means $-2 \times -2 \times -2 \times -2 \times -2$.

Similarly,

$\sqrt{2}^4$ means __________________________.

$\left(\frac{-1}{2}\right)^5$ means __________________________.

π^2 means __________________________.

Fill in the blanks.

Since all variables represent real numbers, powers with variable bases and positive exponents have the same meaning. For example,

y^4 means $y \times y \times y \times y$.

$(3mn)^3$ means $(3mn)(3mn)(3mn)$.

Similarly,

$(-2a)^5$ means ____________________.

$\left(\frac{2a}{b}\right)^4$ means ____________________.

$(-x^2y)^3$ means ____________________.

Any power that has a base which is a real number and an exponent that is negative integer is called a NEGATIVE INTEGRAL POWER. Give examples of four negative integral powers.

$\left(\frac{3}{4}\right)^{-5}$________, __________, __________, __________

In lesson 7, we defined the negative integral power a^{-m} to be the reciprocal of a^{m}.

Negative Exponent
$a^{-m} = \frac{1}{a^{m}}$

Powers with variable bases and negative exponents can be defined in a similar manner. For example,

$$x^{-2} = \frac{1}{x^{2}}$$

$$\frac{1}{y^{-3}} = \frac{1}{\frac{1}{y^{3}}} = 1 \div \frac{1}{y^{3}} = \frac{1}{1} \times \frac{y^{3}}{1} = y^{3}$$

$$\frac{5m^{-5}}{2n^{-3}} = \frac{5 \times \frac{1}{m^{5}}}{2 \times \frac{1}{n^{3}}} = \frac{5}{m^{5}} \div \frac{2}{n^{3}} = \frac{5}{m^{5}} \times \frac{n^{3}}{2} = \frac{5n^{3}}{2m^{5}}$$

Write the following expressions with positive exponents only.

1. $y^{-5} = \frac{1}{y^{\_\_}}$

2. $\frac{6}{x^{-4}} = \frac{6}{\frac{1}{x^{\_\_}}} = \frac{6}{1} \div \frac{1}{x^{\_\_}} = \frac{6}{1} \times \frac{x^{\_\_}}{1} = 6x^{\_\_}$

3. $4x^{-2}y^{3} = \frac{4}{1} \times \frac{1}{x^{\_\_}} \times \frac{y^{3}}{1} = \frac{4y^{3}}{x^{\_\_}}$

Self-correcting Exercise #8

Answers may be found on page 49 of this lesson.

1. Decide whether each of the following statements is true or false.

True or False?

(a) $\left(\frac{-3a}{b}\right)^3 = \frac{-3a}{b} \times \frac{-3a}{b} \times \frac{-3a}{b}$ __________

(b) $x^{-12} = \frac{1}{-x^{12}}$ __________

(c) $4y^{-5} = \frac{1}{4y^5}$ __________

(d) $3abc^{-1} = \frac{3ab}{c}$ __________

(e) $\frac{1}{a^{-3}} = -3a$ __________

(f) $\left(\frac{x}{y}\right)^{-1} = \frac{y}{x}$ __________

(g) $(3b)^{-6} = \frac{1}{3b^6}$ __________

(h) $\frac{1}{x^{-1} + y^{-1}} = x + y$ __________

(i) $\frac{x^{-3}}{2y^{-2}} = \frac{2y^2}{x^3}$ __________

(j) $\frac{3}{mn^{-5}} = \frac{3n^5}{m}$ __________

2. Write with positive exponents.

(a) $m^{-4} =$

(b) $\frac{1}{s^{-7}} =$

(c) $4x^{-3} =$

(d) $\frac{r^{-5}}{s^{-6}} =$

(e) $\frac{x^6}{3y^{-2}} =$

(f) $\frac{2x^{-2}y^3}{5z^{-5}} =$

B. Using the Power Properties

On pages 20 and 25 of this lesson, you were shown how the Product of Powers Property and Quotient of Powers Property can be used to find the product or quotient of powers with like variable bases.

The other power properties that were covered in Lesson 7 can also be applied to powers with variable bases.

The Power of a Power Property tells us how to write a power of a power using a single exponent. This can be done by retaining the same base and multiplying the exponents.

Power of a Power Property
$(a^m)^n = a^{mn}$

Since variables represent real numbers, the Power of a Power Property can be used to simplify powers with variable bases. For example,

$$(x^3)^5 = x^{3 \times 5} = x^{15}$$

$$\left[(-y)^5\right]^{-2} = (-y)^{5 \times -2} = (-y)^{-10}$$

$$\frac{1}{(b^3)^4} = \frac{1}{b^{3 \times 4}} = \frac{1}{b^{12}}$$

Write each of the following powers with a single exponent.

1. $(y^{-3})^8 = y^{\_\times\_} = y^{\_}$

2. $\dfrac{1}{(x^{-5})^4} = \dfrac{1}{x^{\_\times\_}} = \dfrac{1}{x^{\_}}$

3. $(m^2)^3 =$

4. $5(a^2)^3 = 5a^{\_\times\_} = 5a^{\_}$

5. $\left[(-r)^{-4}\right]^2 =$

The Power of a Product Property tells us how the exponent of a power can be applied to every factor in the base.

Power of a Product Property
$(a \times b)^m = a^m \times b^m$

Since variables represent real numbers, the Power of a Product Property can be applied to powers with variable bases. For example,

$$(3xy)^2 = (3)^2(x)^2(y)^2 = 9x^2y^2$$

$$(-2m)^3 = (-2)^3(m)^3 = -8m^3$$

Use the Power of a Product Property to simplify the following expressions.

1. $(-4ab)^3 = (-4)^3\ a^{\_}b^{\_} =$ ________

2. $(9x)^2 = 9^2\_\_ =$ ________

3. $(2xyz)^5 = \_^5\ \_^5\ \_^5\ \_^5 =$ ________

The Power of a Quotient Property tells us how the exponent of a power can be applied to both the numerator and denominator of a fractional base.

Power of a Quotient Property
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Since variables represent real numbers, the Power of a Quotient Property can be applied to powers with variable bases. For example,

$$\left(\frac{x}{y}\right)^7 = \frac{x^7}{y^7}$$

$$\left(\frac{-ab}{c}\right)^5 = \frac{(-ab)^5}{c^5}$$

Use the Power of a Quotient Property to simplify the following expressions.

1. $\left(\frac{3}{x}\right)^4 = \frac{\_\_^4}{\_\_^4} = \frac{81}{\_\_^4}$

2. $\left(\frac{-c}{b}\right)^7 = \frac{(-c)^{\_\_}}{b^{\_\_}}$

3. $\left(\frac{xy}{z}\right)^6 = \frac{(\quad)^{\_\_}}{z^{\_\_}}$

In simplifying some powers with variable bases, you must use more than one power property.

EXAMPLE 1: Simplify the power $(2x^2yz^3)^3$.

<u>Solution</u>

First, the Power of a Product Property can be used to apply the exponent 3 to each factor in the product $2x^2yz^3$.

$$(2x^2yz^3)^3 = 2^3(x^2)^3y^3(z^3)^3$$

Then, the Power of a Power Property can be used to write each factor with a single exponent

$$= 2^3\ x^{2\times3}\ y^3\ z^{3\times3}$$

$$= \underline{\underline{8x^6y^3z^9}}$$

Using this same method, simplify the power $(-3a^3b^2)^4$.

$$(-3a^3b^2)^4 = (-3)^4(a^3)^{\underline{\quad}}(b^2)^{\underline{\quad}} = (-3)^4(a^{\underline{\quad}\times\underline{\quad}})(b^{\underline{\quad}\times\underline{\quad}}) = \underline{\quad}\,a^{\underline{\quad}}b^{\underline{\quad}}$$

EXAMPLE 2: Simplify the power $\left(\frac{2x^2}{-3y^3}\right)^4$.

Solution

First, the Power of a Quotient Property can be used to apply the exponent 4 to both the numerator and denominator of the base.

$$\left(\frac{2x^2}{-3y^3}\right)^4 = \frac{(2x^2)^4}{(-3y^3)^4}$$

Next, the Power of a Product Property can be used to apply the exponent 4 to all the factors in the numerator and denominator.

$$= \frac{2^4(x^2)^4}{(-3)^4(y^3)^4}$$

In the final step, the Power of a Power Property can be used to write each variable factor with a single exponent.

$$= \frac{16x^{2\times 4}}{81y^{3\times 4}}$$

$$= \frac{16x^8}{81y^{12}}$$

Using this same method, simplify the power $\left(\frac{-5x^3}{7z^2}\right)^2$.

$$\left(\frac{-5x^3}{7z^2}\right)^2 = \frac{(-5x^3)^2}{(7z^2)^{\underline{\quad}}} = \frac{(-5)^2(x^3)^{\underline{\quad}}}{7^{\underline{\quad}}(z^2)^{\underline{\quad}}} = \frac{25x^{\underline{\quad}\times\underline{\quad}}}{\underline{\quad}\,z^{\underline{\quad}\times\underline{\quad}}} = \frac{25x^{\underline{\quad}}}{\underline{\quad}\,z^{\underline{\quad}}}$$

Self-correcting Exercise #9

Answers may be found on page 50 of this lesson.

1. Simplify the following powers.

(a) $(5b)^2 =$

(b) $(-3xy)^3 =$

(c) $\left(\frac{5a^5}{-3}\right)^4 =$

(d) $\left(\frac{-9xy^5}{5z^2}\right)^2 =$

2. Use the Power Properties to simplify the following expressions.

(a) $\frac{m^4 \times m^2}{m^3 \times m} =$

(b) $\frac{(a^2b^3)^2}{a \times b^8} =$

(c) $\left(\frac{x}{y}\right)^3 \times (x^3y^2)^2 =$

(d) $\frac{(x^{-2})^3 \times (x^5)^3}{(x^{-3})^2} =$

EXERCISE - Powers of Variable Terms

1. Fill in the blanks.

(a) The power $(a + b)^5$ tells us that ____________________ must be used as a factor __________ times.

(b) The ______________ of ______________ Property allows us to write the product $x^4 \times x^3$ as the single power x^7.

(c) The Power of a Product Property allows us to write the power $(xy)^2$ as the product __________.

(d) y^{-4} is defined to be the reciprocal of ___ and is equal to ___.

(e) $\left(\frac{1}{x}\right)^{-2}$ is equivalent to the power __________.

2. In Column I, some Properties for operating with powers are listed. Choose the property from Column I that was used to arrive at each true statement in Column II.

Column I		Column II
A. Product of Powers Property	_____	1. $(5x)^0 = 1$
B. Power of a Product Property	_____	2. $\frac{x^3}{x^5} = \frac{1}{x^2}$
C. Power of a Power Property	_____	3. $\left(\frac{x}{y}\right)^5 = \frac{x^5}{y^5}$
D. Zero Exponent	_____	4. $\frac{(x^2)^5}{(y^3)^5} = \frac{x^{10}}{y^{15}}$
E. Quotient of Powers Property	_____	5. $x^5 \times x^0 = x^5 \times 1$
F. Power of a Quotient Property	_____	6. $(5xy)^3 = 125x^3y^3$
	_____	7. $(a^3)^4 = a^{12}$
	_____	8. $(abc)^0 = 1$
	_____	9. $3a^2 \times a^4 = 3a^6$
	_____	10. $\frac{x^4}{x} = x^3$

3. Write the following expressions using exponential notation.

(a) $x \times y \times y \times z \times z \times y \times x =$ $x^2y^3z^2$

(b) $a \times a \times a \times a \times b \times b \times b \times b \times b \times c \times c \times c =$ __________

(c) $\frac{5 \times x \times x \times y \times y}{7 \times z \times z \times z} =$ __________

(d) $\frac{(t \times t \times t \times t \times t \times t)}{s \times s} - (r \times r \times r) =$ __________

(e) $4 \times d \times d \times \frac{1}{c} \times \frac{1}{c} =$ __________

4. Simplify the following powers.

(a) $(4y)^2 =$

(b) $(-2b)^3 =$

(c) $(-5x^2y)^3 =$

(d) $(-9x^4y^2)^2 = (-9)^2(x^4)^2(y^2)^2 =$

(e) $(-3x^4y^3z)^4 =$

(f) $\left(\frac{-2x}{y}\right)^2 =$

(g) $\left(\frac{3}{2b}\right)^3 =$

(h) $\left(\frac{-3m^2}{n^3}\right)^5 =$

(i) $\left(\frac{-2ab^2}{c^2}\right)^6 =$

5. Use the power properties to simplify the following expressions. (Answers need not be written with positive exponents.)

(a) $(abc)^{-2}(bc)^{-5} = (a^{-2}b^{-2}c^{-2})(b^{-5}c^{-5}) =$ ________

(b) $\frac{(ab)^5}{a^2b^3} =$

(c) $(-3xy)^2 - 12x^2y^2 =$

(d) $\frac{(x^2)^5}{(x^3)^2} =$

(e) $\dfrac{x^{13}}{x^4 \times (x^2)^3}$ =

(f) $\left(\dfrac{x}{y}\right)^{-3} \times \dfrac{y^5}{x}$ =

(g) $(a^{-2})^{-3} \times (a^4)^{-2}$ =

6. Write each expression with positive exponents only.

(a) $3x^{-5}$ =

(b) $\dfrac{9}{x^{-2}}$ =

(c) $\dfrac{x^2}{4y^{-5}} = \dfrac{x^2}{4 \times \dfrac{1}{y^5}}$ =

(d) $a^{-3} + b^{-3}$ =

(e) $\dfrac{10}{cd^{-1}}$ =

(f) $4m^{-3}n$ =

(g) $5a^2b^{-1}$ =

(h) $\dfrac{3x^{-5}}{2y^{-3}}$ =

(i) $\dfrac{3a^2b^{-4}}{c}$ =

Key to Self-correcting Exercises in Lesson 8

Exercise #1, page 4

1. (a) When $x = -2$, $-5x^3 = -5(-2)^3 = -5(-8) = \underline{40}$

(b) When $x = 0$, $-5x^3 = -5(0)^3 = -5(0) = \underline{0}$

(c) When $x = 2$, $-5x^3 = -5(2)^3 = -5(8) = \underline{-40}$

2. (a)
$$\begin{aligned} & 5x + 1 \\ &= 5(-3) + 1 \\ &= -15 + 1 \\ &= \underline{-14} \end{aligned}$$

(b)
$$\begin{aligned} & 5(x + 1) \\ &= 5(-3 + 1) \\ &= 5(-2) \\ &= \underline{-10} \end{aligned}$$

(c)
$$\begin{aligned} & -2y^2 + 7 \\ &= -2(2)^2 + 7 \\ &= -2(4) + 7 \\ &= -8 + 7 \\ &= \underline{-1} \end{aligned}$$

(d)
$$\begin{aligned} & -4(m + n)^2 \\ &= -4(-8 + 3)^2 \\ &= -4(-5)^2 \\ &= -4(25) \\ &= \underline{-100} \end{aligned}$$

(e)
$$\begin{aligned} & -2\sqrt{a + b - c} \\ &= -2\sqrt{9 + (-2) - 3} \\ &= -2\sqrt{7 - 3} \\ &= -2\sqrt{4} \\ &= -2(2) \\ &= \underline{-4} \end{aligned}$$

(f)
$$\begin{aligned} & 7x^2y - 5z^3 \\ &= 7(2)^2(-1) - 5(-2)^3 \\ &= 7(4)(-1) - 5(-8) \\ &= -28 - (-40) \\ &= -28 + 40 \\ &= \underline{12} \end{aligned}$$

Exercise #2, page 6

1. (a) $2b^2$, $-3c$, $8bc$, -5 (four terms)

(b) x, $\frac{y}{z}$, $-3xz$ (three terms)

(c) $2(b - c)$, $3d$, $-4(m + n)$ (three terms)

(d) $\frac{2(x + 3)}{7}$ (one term)

(e) a, $\frac{-3b}{4}$, $\frac{a + b}{2}$, $\frac{b^2 - a}{b}$ (four terms)

Exercise #3, page 8

1. (a) x, x, $(x - y)$, $(x - y)$

(b) 9, a, a, b, c, c

(c) 5, x, y, $\frac{1}{z}$

(d) 3, $(m + n)$, $\frac{1}{m}$, $\frac{1}{(m - n)}$

(e) a, a, $\frac{1}{3}$, $\frac{1}{b}$, $\frac{1}{c}$

Exercise #4, page 9

1. (a) -7, x^2yz^2 (d) $\frac{-1}{2}$, x

   (b) $\frac{-3}{5}$, a (e) $\frac{12}{5}$, y^2

   (c) 35, a (f) 1, a^2bc^3

2. (a) yes (b) no (c) no

   (d) yes (e) no (f) yes

Exercise #5, page 18

1. (a) $-x$ (b) $-a - b$ (c) y (d) $a - b + c^2$

   (e) $-a^2 + 2ab - b^2$ (f) $a^2 + b^2 - bc - 1$

2. (a)
$$\begin{aligned}&(5a - 6) - (3a - 2)\\ &= (5a - 6) + (-3a + 2)\\ &= (5a - 3a) + (-6 + 2)\\ &= \underline{2a - 4}\end{aligned}$$

   (b)
$$\begin{aligned}&(5x + 2y + 3z) - (-x + 2y - 5z)\\ &= (5x + 2y + 3z) + (x - 2y + 5z)\\ &= (5x + x) + (2y - 2y) + (3z + 5z)\\ &= \underline{6x + 8z}\end{aligned}$$

   (c)
$$\begin{aligned}&(3x^2 - 2y^2 - 4xy) - (7y^2 - xy)\\ &= (3x^2 - 2y^2 - 4xy) + (-7y^2 + xy)\\ &= 3x^2 + (-2y^2 - 7y^2) + (-4xy + xy)\\ &= \underline{3x^2 - 9y^2 - 3xy}\end{aligned}$$

   (d)
$$\begin{aligned}&(2x - y + 3) - (-2y - 5)\\ &= (2x - y + 3) + (2y + 5)\\ &= 2x + (-y + 2y) + (3 + 5)\\ &= \underline{2x + y + 8}\end{aligned}$$

3. (a)
$$\begin{aligned}&6b^2 + 2b^2 - 9b^2\\ &= 8b^2 - 9b^2\\ &= \underline{-b^2}\end{aligned}$$

   (b)
$$\begin{aligned}&\sqrt{2}a + 5\sqrt{2}a\\ &= (\sqrt{2} + 5\sqrt{2})\ a\\ &= \underline{6\sqrt{2}a}\end{aligned}$$

   (c)
$$\begin{aligned}&3a^2 + 2a - a^2 + 5a - 4a^2\\ &= (3a^2 - a^2 - 4a^2) + (2a + 5a)\\ &= (3a^2 - 5a^2) + 7a\\ &= \underline{-2a^2 + 7a}\end{aligned}$$

   (d)
$$\begin{aligned}&2s + 3r - s - r\\ &= (2s - s) + (3r - r)\\ &= \underline{s + 2r}\end{aligned}$$

(e) $5a^4 - 9a^3 - 12a^2 - 5a^3 + 5a^2 - 6a^4 - 3a^4 + 2a^3 - 6a^2$

$= (5a^4 - 6a^4 - 3a^4) + (-9a^3 - 5a^3 + 2a^3) + (-12a^2 + 5a^2 - 6a^2)$

$= (5a^4 - 9a^4) + (-14a^3 + 2a^3) + (-18a^2 + 5a^2)$

$= \underline{-4a^4 - 12a^3 - 13a^2}$

(f) $(2x - y + 3) - (-2y + 5) - (3x - 4y)$

OPERATION HAS BEEN CHANGED TO ADDITION.

$= (2x - y + 3) + \underbrace{(2y - 5)}_{\text{ADDITIVE INVERSE OF } -2y+5} + \underbrace{(-3x + 4y)}_{\text{ADDITIVE INVERSE OF } 3x-4y.}$

$= (2x - 3x) + (-y + 2y + 4y) + (3 - 5)$

$= -x + (-y + 6y) + (-2)$

$= \underline{-x + 5y - 2}$

(g) $(3a + b - 5c) + (2a - 5b + 4c) - (3a - 2b - 5c)$

$= (3a + b - 5c) + (2a - 5b + 4c) \underset{\uparrow}{+} \underbrace{(-3a + 2b + 5c)}_{\text{ADDITIVE INVERSE OF } 3a-2b-5c}$

CHANGE OPERATION

$= (3a + 2a - 3a) + (b - 5b + 2b) + (-5c + 4c + 5c)$

$= (5a - 3a) + (3b - 5b) + (9c - 5c)$

$= \underline{2a - 2b + 4c}$

Exercise #6, page 23

1.

	Product of Numerical Factors	Product of x Factors	Product of y Factors	Entire Product
$(5x^2y)(-10x^7y^3)$	$5 \times -10 = \underline{-50}$	$x^2 \times x^7 = \underline{x^9}$	$y \times y^3 = \underline{y^4}$	$\underline{-50x^9y^4}$
$(-2x^3y^3)(7x^{-5}y^2)$	$-2 \times 7 = \underline{-14}$	$x^3 \times x^{-5} = \underline{x^{-2}}$	$y^3 \times y^2 = \underline{y^5}$	$\underline{-14x^{-2}y^5}$
$\left(\frac{1}{3}xy\right)(-6x^4y^{-3})$	$\frac{1}{3} \times -6 = \underline{-2}$	$x \times x^4 = \underline{x^5}$	$y \times y^{-3} = \underline{y^{-2}}$	$\underline{-2x^5y^{-2}}$
$(-xy)(5xy)$	$-1 \times 5 = \underline{-5}$	$x \times x = \underline{x^2}$	$y \times y = \underline{y^2}$	$\underline{-5x^2y^2}$
$(6x^2y)(-3xy^3)$	$6 \times -3 = \underline{-18}$	$x^2 \times x = \underline{x^3}$	$y \times y^3 = \underline{y^4}$	$\underline{-18x^3y^4}$

2. (a) $x^5 \times x^4 = x^{5+4} = \underline{x^9}$

(b) $(-3a^3)(7a^5) = (-3 \times 7)(a^3 \times a^5) = \underline{-21a^8}$

(c) $(3b)(-5b^4x) = (3 \times -5)(b \times b^4)(x) = \underline{-15b^5x}$

(d) $\left(\frac{1}{2}xy\right)(-2xy^3) = \left(\frac{1}{2} \times -2\right)(x \times x)(y \times y^3) = \underline{-x^2y^4}$

(e) $(-4m^2n^3)(mn^4)(-2m^3n) = (-4 \times 1 \times -2)(m^2 \times m \times m^3)(n^3 \times n^4 \times n) = \underline{8m^6n^8}$

(f) $(6cd)(-5c^2)(3d^2) = (6 \times -5 \times 3)(c \times c^2)(d \times d^2) = \underline{-90c^3d^3}$

(g) $(-5a^2b^3c)(7ac^5) = (-5 \times 7)(a^2 \times a)(b^3)(c \times c^5) = \underline{-35a^3b^3c^6}$

(h) $(-x^2y^2z^3)(3xy^3) = (-1 \times 3)(x^2 \times x)(y^2 \times y^3)(z^3) = \underline{-3x^3y^5z^3}$

(i)
$$\begin{aligned} & 2a(a^2 - ab + b^2) \\ = & (2a)(a^2) + (2a)(-ab) + (2a)(b^2) \\ = & \underline{2a^3 - 2a^2b + 2ab^2} \end{aligned}$$

(j)
$$\begin{aligned} & -3x(y^2 + y - 7) \\ = & (-3x)(y^2) + (-3x)(y) + (-3x)(-7) \\ = & \underline{-3xy^2 - 3xy + 21x} \end{aligned}$$

(k)
$$\begin{aligned} & (2a + b)(3a^2 + 5) \\ = & (2a)(3a^2) + (b)(3a^2) + (2a)(5) + (b)(5) \\ = & \underline{6a^3 + 3a^2b + 10a + 5b} \end{aligned}$$

(l)
$$\begin{aligned} & (2x + 5)(3x - 2) \\ = & (2x)(3x) + (5)(3x) + (2x)(-2) + (5)(-2) \\ = & 6x^2 + 15x - 4x - 10 \\ = & \underline{6x^2 + 11x - 10} \end{aligned}$$

(m)
$$\begin{aligned} & (a^2 - 2ab)(a^2 + 2ab) \\ = & (a^2)(a^2) + (-2ab)(a^2) + (a^2)(2ab) + (-2ab)(2ab) \\ = & a^4 - 2a^3b + 2a^3b - 4a^2b^2 \\ = & \underline{a^4 - 4a^2b^2} \end{aligned}$$

Exercise #7, page 27

1.

	Quotient of Numerical Factors	Quotient of x Factors	Quotient of y Factors	Entire Quotient
$\frac{27xy^9}{-3x^4y^4}$	$\frac{27}{-3} = -9$	$\frac{x}{x^4} = \frac{1}{x^3}$	$\frac{y^9}{y^4} = y^5$	$\frac{-9y^5}{x^3}$
$\frac{36x^4y^5}{9x^5y^2}$	$\frac{36}{9} = 4$	$\frac{x^4}{x^5} = \frac{1}{x}$	$\frac{y^5}{y^2} = y^3$	$\frac{4y^3}{x}$
$\frac{3x^4y^3}{\frac{1}{2}x^2y}$	$\frac{3}{\frac{1}{2}} = 6$	$\frac{x^4}{x^2} = x^2$	$\frac{y^3}{y} = y^2$	$6x^2y^2$
$\frac{-48x^6y^5}{-16x^9y^4}$	$\frac{-48}{-16} = 3$	$\frac{x^6}{x^9} = \frac{1}{x^3}$	$\frac{y^5}{y^4} = y$	$\frac{3y}{x^3}$
$\frac{5xy}{25x^3y^4}$	$\frac{5}{25} = \frac{1}{5}$	$\frac{x}{x^3} = \frac{1}{x^2}$	$\frac{y}{y^4} = \frac{1}{y^3}$	$\frac{1}{5x^2y^3}$

2. (a) $\frac{x^4}{x^2} = x^{4-2} = x^2$

(b) $\frac{m}{m^5} = \frac{1}{m^{5-1}} = \frac{1}{m^4}$

(c) $\frac{15y^3}{-5y^2} = \frac{15}{-5} \times \frac{y^3}{y^2} = -3y$

(d) $\frac{-2a}{4a^2} = \frac{-2}{4} \times \frac{a}{a^2} = \frac{-1}{2} \times \frac{1}{a} = \frac{-1}{2a}$

(e) $\frac{-12a^2b^2c^4}{-8b^2c^5}$

$= \frac{-12}{-8} \times a^2 \times \frac{b^2}{b^2} \times \frac{c^4}{c^5}$

$= \frac{3}{2} \times a^2 \times 1 \times \frac{1}{c}$

$= \frac{3a^2}{2c}$

(f) $\frac{-20m^2}{45m^3n}$

$= \frac{-20}{45} \times \frac{m^2}{m^3} \times \frac{1}{n}$

$= \frac{-4}{9} \times \frac{1}{m} \times \frac{1}{n}$

$= \frac{-4}{9mn}$

Exercise #8, page 35

1. (a) true; $\frac{-3a}{b}$ is used as a factor three times.

(b) false; $x^{-12} = \frac{1}{x^{12}}$

(c) false; $4y^{-5} = 4 \times \frac{1}{y^5} = \frac{4}{y^5}$

(d) true; $3abc^{-1} = 3ab \times \frac{1}{c} = \frac{3ab}{c}$

(e) false; $\frac{1}{a^{-3}} = \frac{1}{\frac{1}{a^3}} = a^3$

(f) true; $\left(\frac{x}{y}\right)^{-1} = \frac{1}{\frac{x}{y}} = \frac{y}{x}$

(g) false; $(3b)^{-6} = \frac{1}{(3b)^6}$

(h) false; $\frac{1}{x^{-1} + y^{-1}} = \frac{1}{\frac{1}{x} + \frac{1}{y}}$

(i) false; $\frac{x^{-3}}{2y^{-2}} = \frac{\frac{1}{x^3}}{2 \times \frac{1}{y^2}} = \frac{1}{x^3} \div \frac{2}{y^2} = \frac{1}{x^3} \times \frac{y^2}{2} = \frac{y^2}{2x^3}$

(j) true; $\frac{3}{mn^{-5}} = \frac{3}{m \times \frac{1}{n^5}} = \frac{3}{1} \div \frac{m}{n^5} = \frac{3}{1} \times \frac{n^5}{m} = \frac{3n^5}{m}$

2. (a) $m^{-4} = \frac{1}{m^4}$

(b) $\frac{1}{s^{-7}} = \frac{1}{\frac{1}{s^7}} = 1 \div \frac{1}{s^7} = 1 \times \frac{s^7}{1} = s^7$

(c) $4x^{-3} = 4 \times \frac{1}{x^3} = \frac{4}{x^3}$

(d) $\frac{r^{-5}}{s^{-6}} = \frac{\frac{1}{r^5}}{\frac{1}{s^6}} = \frac{1}{r^5} \div \frac{1}{s^6} = \frac{1}{r^5} \times \frac{s^6}{1} = \frac{s^6}{r^5}$

(e) $\frac{x^6}{3y^{-2}} = \frac{x^6}{\frac{3}{1} \times \frac{1}{y^2}} = \frac{x^6}{1} \div \frac{3}{y^2} = \frac{x^6}{1} \times \frac{y^2}{3} = \frac{x^6y^2}{3}$

(f) $\frac{2x^{-2}y^3}{5z^{-5}} = \frac{\frac{2}{1} \times \frac{1}{x^2} \times \frac{y^3}{1}}{\frac{5}{1} \times \frac{1}{z^5}} = \frac{2y^3}{x^2} \div \frac{5}{z^5} = \frac{2y^3}{x^2} \times \frac{z^5}{5} = \frac{2y^3z^5}{5x^2}$

Exercise #9, page 40

1. (a) $(5b)^2 = 5^2b^2 = 25b^2$

(b) $(-3xy)^3 = (-3)^3x^3y^3 = -27x^3y^3$

(c) $\left(\frac{5a^5}{-3}\right)^4 = \frac{(5a^5)^4}{(-3)^4} = \frac{5^4(a^5)^4}{(-3)^4} = \frac{625a^{5\times4}}{81} = \frac{625a^{20}}{81}$

(d) $\left(\frac{-9xy^5}{5z^2}\right)^2 = \frac{(-9xy^5)^2}{(5z^2)^2} = \frac{(-9)^2x^2(y^5)^2}{5^2(z^2)^2} = \frac{81x^2y^{5\times2}}{25z^{2\times2}} = \frac{81x^2y^{10}}{25z^4}$

ADD EXPONENTS. SUBTRACT EXPONENTS.

2. (a) $\frac{m^4 \times m^2}{m^3 \times m} = \frac{m^{4+2}}{m^{3+1}} = \frac{m^6}{m^4} = m^{6-4} = m^2$

(b) $\frac{(a^2b^3)^2}{a \times b^8} = \frac{(a^2)^2(b^3)^2}{a \times b^8} = \frac{a^4b^6}{a \times b^8} = \frac{a^4}{a} \times \frac{b^6}{b^8} = \frac{a^3}{1} \times \frac{1}{b^2} = \frac{a^3}{b^2}$

(c) $\left(\frac{x}{y}\right)^3 \times (x^3y^2)^2 = \frac{x^3}{y^3} \times (x^3)^2(y^2)^2 = \frac{x^3}{y^3} \times \frac{x^6}{1} \times \frac{y^4}{1} = (x^3 \times x^6) \times \frac{y^4}{y^3} = x^9y$

(d) $\frac{(x^{-2})^3 \times (x^5)^3}{(x^{-3})^2} = \frac{x^{-2\times3} \times x^{5\times3}}{x^{-3\times2}} = \frac{x^{-6} \times x^{15}}{x^{-6}} = \frac{x^9}{x^{-6}} = x^{9-(-6)} = x^{15}$

End of Lesson 8

Lesson 9

Using Algebra

Basic Algebra and Geometry

USING ALGEBRA

Topic One: Mathematical Phrases

A mathematical phrase involves number symbols, letter symbols, and operation symbols. It tells us what operations are performed on certain known and unknown numbers. For example, the mathematical phrase $5x - 7$ tells us that an unknown number is multiplied by 5 and the result is decreased by 7.

When using algebra to solve problems, you must be able to recognize English expressions that can be translated into the operation symbols $+$, $-$, $\times$, $\div$.

1. Such phrases as "the sum of", "added to", "increased by" may be translated into the addition symbol, "+".
   EXAMPLES:

English phrase	Mathematical phrase
the <u>sum</u> of y and 2	$y + 2$
8 is <u>added to</u> x	$x + 8$
5 is <u>increased by</u> n	$5 + n$

   How would you write "the sum of r and 5" in symbols? ________

2. Such phrases as "subtracted from", "the difference of", "less than", and "decreased by" may be translated into the subtraction symbol, "-".
   EXAMPLES:

English phrase	Mathematical phrase
3 is <u>subtracted from</u> x	$x - 3$
the <u>difference of</u> x and y	$x - y$
2 is <u>decreased by</u> x	$2 - x$
7 <u>less than</u> n	$n - 7$

This means that n is decreased by 7.

   How would you write "the difference of x and 2" in symbols? ________

3. Such phrases as "the product of", "times", and "multiplied by" may be translated into the multiplication symbol, "×".
EXAMPLES:

English phrase	Mathematical phrase
the product of x and y	xy
3 times n	3n
x is multiplied by 3 (or x is tripled)	3x

How would you write "y is multiplied by -6" in symbols? __________

4. Such phrases as "divided by" or "the quotient of" can be translated into the division symbol, "÷".
EXAMPLES:

English phrase	Mathematical phrase
x is divided by 3	$x \div 3$ (or $\frac{x}{3}$)
the quotient of x and y	$x \div y$ (or $\frac{x}{y}$)

How would you write "the quotient of m and 2" in symbols? __________

EXERCISE - Mathematical Phrases

1. Decide what operation is suggested by each phrase. (Use addition, subtraction, multiplication, or division.)

(a) product of multiplication

(b) plus __________

(c) decreased by __________

(d) divided by __________

(e) quotient of __________

(f) difference of __________

(g) increased by __________

(h) times __________

(i) sum of __________

2. Write a mathematical phrase for each English phrase. (In each case, use n for the unknown number.)

(a) A certain number is increased by 5. $n+5$

(b) A certain number is divided by 2. ________

(c) A certain number is doubled. ________

(d) A certain number is decreased by 7. ________

(e) 5 times a certain number. ________

(f) The product of a certain number and 7. ________

(g) The quotient of a certain number and 6. ________

(h) A certain number is subtracted from 8. ________

(i) 3 is added to a certain number. ________

(j) 5 is divided by a certain number. ________

(k) 8 less than a certain number. ________

A. Building Mathematical Phrases

We can build a mathematical phrase, step by step, from a set of instructions.

EXAMPLE: Build a phrase by following the instructions that you are to: "Take a number, multiply by three, and then add two."

Solution

Instruction	Phrase
Take a number.	n
Multiply by three.	3n
Add two.	3n + 2

In most cases in which you are called upon to build a mathematical phrase, the instructions are not spelled out quite so precisely. You must sort out the instructions yourself and decide in what order the operations are to be performed upon the unknown.

EXAMPLE: Translate the English phrase "5 is added to 4 times a certain number" to a mathematical phrase.

Solution

First, we must multiply the unknown number by 4 and then add 5 to the result.

Instruction	Phrase
Take a number.	n
Multiply it by 4.	4n
Add 5.	4n + 5

In this case, the corresponding mathematical phrase is 4n + 5.

EXERCISE - Building Mathematical Phrases

1. Build each mathematical phrase by writing out the step-by-step instructions you must follow.

(a) Three less than the quotient of a certain number and 5

Instruction	Phrase
Take a number.	n
Divide by 5.	$\frac{n}{5}$
Subtract 3.	$\frac{n}{5} - 3$

(b) Twice a number increased by 9

Instruction	Phrase
Take a number.	

(c) Five less than four times a number

Instruction	Phrase
Subtract 5.	

(d) A certain number is divided by 4 and the result increased by 2.

Instruction	Phrase
______	______
Divide by 4.	______
______	______

(e) Five is added to one-half a number.

Instruction	Phrase
______	______
______	$\frac{1}{2}n$
______	______

(f) A certain number is tripled and then divided by 4.

Instruction	Phrase
______	______
______	______
______	______

2. Write a mathematical phrase for each statement.

(a) Eight less than five times a number ______

(b) Six is divided by the product of 3 and a certain number. $6 \div$ ______

(c) Six times a number is increased by 18. ______

(d) Two-thirds of a number is decreased by 5. ______

(e) 8 is increased by the quotient of a certain number into 3. ______

(f) The sum of 4 and a certain number is divided by 9. ______

B. Taking Apart Mathematical Phrases

In taking apart a mathematical phrase, we simply reverse the procedure we used in building it. Our object is to break up the phrase until we end up with the variable alone.

A knowledge of taking apart mathematical phrases is useful in solving equations. In solving equations, we are concerned with taking apart mathematical phrases so that the variable is isolated on one side of the equation.

EXAMPLE: Describe how you could take apart the mathematical phrase $3n + 2$ so that you end up with only n.

Solution

Instruction	Phrase
Take a phrase	$3n + 2$
Subtract 2	$3n$
Divide by 3	n

Note that in undoing this phrase, we had to use the idea of inverse operations. Remember that addition and subtraction are inverse operations, while multiplication and division are inverse operations. In order to undo the operation of adding 2 to 3n, we subtract 2 from the given phrase. In order to undo the operation of multiplying n by 3, we divide the given phrase by 3.

EXERCISE - Undoing Mathematical Phrases

1. Supply the instructions for undoing the following mathematical phrases.

	Instruction	Phrase
a.	Take a phrase.	$4y - 5$
	Add 5.	$4y$
	__________	y
b.	Take a phrase.	$5n + 7$
	__________	$5n$
	__________	n

	Instruction	Phrase
c.	Take a phrase.	$\frac{y}{2} - 3$
	______________	$\frac{y}{2}$
	Multiply by 2.	y
d.	Take a phrase.	$\frac{y}{5} + 1$
	______________	$\frac{y}{5}$
	______________	y

2. In each question, the instructions are provided for undoing the given mathematical phrase. Supply the missing phrases.

	Instruction	Phrase
a.	Take a phrase.	$\frac{1}{3}y - 2$
	Add 2.	$\frac{1}{3}y$
	Multiply by 3.	______________
b.	Take a phrase.	$\frac{2x}{3}$
	Multiply by 3.	______________
	Divide by 2.	______________
c.	Take a phrase.	$3y - 6$
	Add 6.	______________
	Divide by 3.	______________
d.	Take a phrase.	$2x + 1$
	Subtract 1.	______________
	Divide by 2.	______________

3. Undo each mathematical phrase by supplying the missing instructions and phrases.

	Instruction	Phrase
a.	Take a phrase.	$3x + 7$
	Subtract 7.	________
	Divide by 3.	________
b.	Take a phrase.	$\frac{1}{2}y - 9$
	________________	________
	________________	________
c.	Take a phrase.	$\frac{3n}{5}$
	________________	________
	________________	________
d.	Take a phrase.	$10 + 3y$
	________________	________
	________________	________
e.	Take a phrase.	$\frac{x}{7} + 5$
	________________	________
	________________	________

Topic Two: Equations

An EQUATION is a statement of equality between numerical or variable expressions. All equations contain the relations symbol, "=". The following are examples of equations.

(1) $5 + 3 = 2 \times 4$
(2) $8 \div 4 = 4 \div 8$
(3) $3x + 5 = 2x - 8$
(4) $x = y - 5$

In an equation, the expression to the left of the equals sign is called the LEFT MEMBER of the equation and the expression to the right of the equals is called the RIGHT MEMBER.

Some equations involve real numbers only and are either true or false. Equations (1) and (2) above are equations of this type.

(1) $5 + 3 = 2 \times 4$ is a number equation whose left side equals 8 and right side equals 8. Since both sides of the equation name the same number, this is a true statement.

(2) $8 \div 4 = 4 \div 8$ is a number equation whose left side equals 2 and right side equals $\frac{1}{2}$. Since the two sides of the equation name different numbers, this is a false statement.

Give examples of three other number equations that are true.

$\frac{1}{2} \times \frac{-6}{1} = -3$, ________________ , ________________

Give examples of three other number equations that are false.

________________ , ________________ , ________________

Decide whether each of the following number equations is true or false.

$3 \times 5 = 5 \times 3$	$13 \times 0 = 13$	$(6 \div 2) + 5 = (2 \div 6) + 5$
true	________	________
$(9 \times 1) + 3 = 13$	$4 + 4 + 4 = 3 \times 4$	$36 = (3 \times 10) + 6$
________	________	________

While some equations involve real numbers only, others involve variables. If an equation contains at least one variable, it is called a CONDITIONAL EQUATION. As it stands, a conditional equation is neither true nor false. It becomes true or false only when specific values have been assigned to the variables involved. Equations (3) and (4) at the top of page 9 of this lesson are conditional equations.

(3) $3x + 5 = 2x - 8$ is a conditional equation involving the one variable x. If we assign the value 5 to x, we obtain the number equation $15 + 5 = 10 - 8$ which is false. On the other hand, if we assign the value -13 to x, we obtain the number equation $-39 + 5 = -26 - 8$ which is true.

(4) $x = y - 5$ is a conditional equation involving the two variables x and y. If we assign the value -3 to x and the value 1 to y, we obtain the number equation $-3 = 1 - 5$ which is false. On the other hand, if we assign the value 3 to x and the value 8 to y, we obtain the number equation $3 = 8 - 5$ which is true.

EXERCISE - Mathematical Equations

1. Classify each of the following equations as being true, false, or conditional.

(a) $6 + 3 = 2 + 7$ true

(b) $4 + x = 6$ ______

(c) $4 + 3 = 6$ ______

(d) $28 \div 4 = y$ ______

(e) $x + y = 12$ ______

(f) $7 \times 9 = 63$ ______

2. For each conditional equation below, replace the variables by the numbers indicated and decide whether the resulting number equation is true or false.

(a) $x - y = -5$
Replace x by -8 and y by -3.

$-8-(-3) = -5$

$-8+3 = -5$

$-5 = -5$

(true)

(b) $7m + 12 = 54$
Replace m by 5.

(c) $3mn - 2 = 7$
Replace m by 2 and n by 3.

(d) $x^2 + 8x = -15$ Replace x by -3.	(e) $x^2 + 8x = -15$ Replace x by -5.	(f) $5x^2 + xy = 8$ Replace x by -1 and y by -3.
(g) $x + 2y = 4$ Replace x by 6 and y by -5.	(h) $2x - 3y = 7$ Replace x by 2 and y by -1.	(i) $2x + 3 = -5x + 17$ Replace x by 2.

Topic Three: Linear Equations in One Variable

In this course, we will deal only with conditional equations that have just one variable and this variable appears to the first power only. Such equations are called LINEAR EQUATIONS IN ONE VARIABLE. (The word "linear" tells us that the variable appears only to the first power.) The following are examples of equations of this type.

THE VARIABLE x IS TO THE FIRST POWER.

$$x + 3 = 9$$

$$2x - 5 = 17$$

$$5x + 2 = 3x - 7$$

$$6x + 3 + 5x = 8x - 7$$

Given a linear equation in one variable, we are interested in finding one value from the domain of the variable which will make the equation a true statement. This value is called the ROOT or SOLUTION of the equation. The subset of the domain which consists of the one value of the variable that makes the equation true is called the SOLUTION SET of the equation.

If the domain of the variable contains only a small number of elements, the solution set of the equation can be determined by testing each element from the domain in the equation. The one element which makes the equation true must belong to the solution set.

EXAMPLE: If the domain of x is $A = \{4, 5, 6\}$, find the solution set of the equation $3x + 2 = 17$.

Solution

Substitute each of the values from the domain of x into the given equation and see if the resulting number equations are true or false.

If $x = 4$,	If $x = 5$,	If $x = 6$,
$3(4) + 2 = 17$	$3(5) + 2 = 17$	$3(6) + 2 = 17$
$12 + 2 = 17$	$15 + 2 = 17$	$18 + 2 = 17$
$14 = 17$	$17 = 17$	$20 = 17$
(false)	(true)	(false)

Since we obtain a true statement when $x = 5$, we can say that 5 is a root of the equation $3x + 2 = 17$ when $x \in \{4, 5, 6\}$. Thus the solution set of the equation for the given domain is $\{5\}$.

For the equation $2n - 5 = 11$, suppose that the domain of n is $\{-3, 5, 8\}$. Test each element from the domain in the equation and decide if the resulting number equation is true or false. Then, state the solution set of the equation.

If $n = -3$,	If $n = 5$,	If $n = 8$,
$2(-3) - 5 = 11$		
___ $- 5 = 11$		
___ $= 11$		
(________) true or false?		

Since we obtain a true statement when n = __, we can say that ___ is a root of the equation and the solution set of the equation is {___} for the given domain.

If the domain of a variable contains an infinite number of elements, it is impossible to test all the values of the variable in the given equation. In such cases, the solution set can often be determined by inspection.

EXAMPLE: If $x \in R$, find the solution set of the equation $2x = -6$.

<u>Solution</u>

$$2x = -6$$

In this case, we are seeking a value of x which when multiplied by 2 will equal -6. The only real number which satisfies this condition is -3.

i.e. when $x = -3$, $2(-3) = -6$
$-6 = -6$
(true)

Since we obtain a true statement when $x = -3$, the solution set of the equation is $\{-3\}$.

Solve each of the following equations by inspection.

1. $4x = 28$ is satisfied when $x =$ 7 since $4 \times$ 7 $= 28$ is a true statement.

2. $15 \div x = 3$ is satisfied when $x =$ ___ since $15 \div$ ___ $= 3$ is a true statement.

3. $n + 7 = 15$ is satisfied when $n =$ ___ since ___ $+ 7 = 15$ is a true statement.

4. $9 - y = 4$ is satisfied when $y =$ ___ since $9 -$ ___ $= 4$ is a true statement.

5. $32n = 160$ is satisfied when $n =$ ___ since $32 \times$ ___ $= 160$ is a true statement.

6. $n \div 7 = 4$ is satisfied when $n =$ ___ since ___ $\div 7 = 4$ is a true statement.

7. $z - 3 = 12$ is satisfied when $z =$ ___ since ___ $- 3 = 12$ is a true statement.

8. $2x + 1 = 7$ is satisfied when $x =$ ___ since $(2 \times$ ___$) + 1 = 7$ is a true statement.

9. $\frac{x}{2} = -1$ is satisfied when $x =$ ___ since $\frac{}{2} = -1$ is a true statement.

10. $3x - 1 = 11$ is satisfied when $x =$ ___ since $(3 \times$ ___$) - 1 = 11$ is a true statement.

Topic Four: Formal Method for Solving Linear Equations in One Variable

The solution set of a linear equation in one variable cannot always be easily determined by inspection. For example, the root of the equation $-4x + 7 = -3x - 9$ is not obvious at first glance. We need a formal method for solving all linear equations in one variable.

A. Maintaining Balance in an Equation

An equation is a very delicately balanced statement. It is like a sensitive scale in balance. A weight added to only one side of the equation will destroy the balance. When working with equations, the balance can be maintained only if the same operation is performed on both sides of the equation.

> In general, you will find that the balance of an equation is maintained if:
>
> 1. The same number is added to both sides.
> 2. The same number is subtracted from both sides.
> 3. Each side is multiplied by the same number (not zero).
> 4. Each side is divided by the same number (not zero).

We can use the four principles above to solve equations. By performing the same operations on both sides of the equation, we can arrive at an equation of the form $x = k$, where k is a number. In order to arrive at an equation of this type, we must undo the mathematical phrase in the equation that contains the variable x. Review pages 6-8 of this lesson where you had some practice undoing mathematical phrases.

1. Use the operation of addition to undo a subtraction.

EXAMPLE: $x - 3 = 9$

In order to undo the operation of subtracting 3 from x, we must add 3 to both sides of the equation.

3 IS ADDED TO BOTH SIDES OF THE EQUATION.

$$x - 3 + 3 = 9 + 3$$
$$x = 12$$

Solve the following equations by adding the same amount to both sides.

$x - 9 = 12$	$x - 1 = 7$	$x - 4 = 3$
$x - 9 + \underline{9} = 12 + \underline{9}$	$x - 1 + \_\_ = 7 + \_\_$	$x - 4 + \_\_ = 3 + \_\_$
$x = \_\_$	$x = \_\_$	$x = \_\_$

2. <u>Use the operation of subtraction to undo an addition.</u>

EXAMPLE: $x + 8 = 5$

In order to undo the operation of adding 8 to x, we must subtract 8 from <u>both</u> sides of the equation.

$$x + 8 - 8 = 5 - 8$$

8 IS SUBTRACTED FROM BOTH SIDES.

$$x = -3$$

Solve the following equations by subtracting the same amount from each side.

$x + 9 = 12$	$x + 4 = 23$	$x + 3 = 1$
$x + 9 - \underline{9} = 12 - \underline{9}$	$x + 4 - \_\_ = 23 - \_\_$	$x + 3 - \_\_ = 1 - \_\_$
$x = \_\_$	$x = \_\_$	$x = \_\_$

3. <u>Use the operation of multiplication to undo a division.</u>

EXAMPLE: $\frac{x}{3} = 12$

In order to undo the operation of dividing x by 3, we must multiply each side of the equation by 3.

$$\cancel{3} \times \frac{x}{\cancel{3}} = 3 \times 12$$

EACH SIDE HAS BEEN MULTIPLIED BY 3.

$$x = 36$$

Solve the following equations by multiplying each side by the same amount.

$\frac{y}{7} = 4$	$\frac{1}{2}x = 7$	$\frac{y}{5} = -2$
$\underline{7} \times \frac{y}{7} = \underline{7} \times 4$	$\_\_ \times \frac{1}{2}x = \_\_ \times 7$	$\_\_ \times \frac{y}{5} = \_\_ \times -2$
$y = \_\_$	$x = \_\_$	$y = \_\_$

4. <u>Use the operation of division to undo a multiplication.</u>

EXAMPLE: $5x = -20$

In order to undo the operation of multiplying x by 5, we must divide each side of the equation by 5.

$$\frac{5x}{5} = \frac{-20}{5}$$ EACH SIDE HAS BEEN DIVIDED BY 5.

$$x = -4$$

Solve the following equations by dividing each side by the same amount.

$3x = 21$ $\qquad$ $-7x = 28$ $\qquad$ $4x = 24$

$\dfrac{3x}{\boxed{3}} = \dfrac{21}{\boxed{3}}$ $\qquad$ $\dfrac{-7x}{-7} = \dfrac{28}{\boxed{}}$ $\qquad$ $\dfrac{4x}{\boxed{}} = \dfrac{24}{\boxed{}}$

$x =$ _____ $\qquad$ $x =$ _____ $\qquad$ $x =$ _____

<u>Self-correcting Exercise #1</u>

Answers to this exercise may be found on page 52 of this lesson.

1. Fill in the blanks.

(a) $-8 + 2 =$ __________

(b) $-7 - 3 =$ __________

(c) $6 - 8 =$ __________

(d) $-13 - 4 =$ __________

(e) $-5 + 4 =$ __________

(f) $5 - 4 =$ __________

(g) $-5 - 4 =$ __________

(h) $3 - 9 =$ __________

(i) $1 - 2 =$ __________

(j) $2 - 5 =$ __________

(k) $3 \times 5 =$ __________

(l) $3 \times -5 =$ __________

(m) $-2 \times -3 =$ __________

(n) $-6 \times 4 =$ __________

(o) $-2 \times -2 =$ __________

(p) $\frac{6}{3} =$ __________

(q) $\frac{-9}{3} =$ __________

(r) $\frac{-6}{3} =$ __________

(s) $\frac{-6}{-3} =$ __________

(t) $\frac{12}{-4} =$ __________

2. Tell what operation was performed on each side of the equation in order to obtain equation (2) from equation (1).

(a) $2x = 8$ (1)

Divide each side by 2.

$x = 4$ (2)

(b) $x + 3 = 12$ (1)

____

$x = 9$ (2)

(c) $\frac{1}{2}x = -3$ (1)

____

$x = -6$ (2)

(d) $x - 3 = -2$ (1)

____

$x = 1$ (2)

(e) $-3x = 90$ (1)

____

$x = -30$ (2)

(f) $x + 7 = 2$ (1)

____

$x = -5$ (2)

3. Solve each of the following equations by performing the same operation on both sides.

(a)
$$x + 35 = 25$$
$$x + 35 - \_\_ = 25 - \_\_$$
$$x = \_\_\_\_$$

(b)
$$x - 12 = -43$$
$$x - \_\_ + \_\_ = -43 + \_\_$$
$$x = \_\_\_\_$$

(c)
$$\frac{x}{9} = -7$$
$$\_\_ \times \frac{x}{9} = \_\_ \times -7$$
$$x = \_\_\_\_$$

(d) $-8x = -48$

EXERCISE - Solving Equations

1. Tell what operation you would perform on each side to solve the equation.

Equation	Operation
(a) $x + 3 = -2$	Subtract 3 from each side.
(b) $x - 3 = 6$	__________
(c) $\frac{x}{6} = 7$	__________
(d) $5x = 10$	__________
(e) $x + 4 = 9$	__________
(f) $\frac{1}{3}x = 9$	__________
(g) $-4x = 16$	Divide each side by -4.
(h) $9 + x = 4$	__________
(i) $x - 6 = 2$	__________

2. Solve the following equations by performing the same operation on each side.

(a)
$$x - 5 = 8$$
$$x - 5 + \underline{5} = 8 + \underline{5}$$
$$x = \underline{\quad}$$

(b)
$$\frac{x}{7} = 3$$
$$\underline{\quad} \times \frac{x}{7} = \underline{\quad} \times 3$$
$$x = \underline{\quad}$$

(c)
$$x + 5 = 1$$
$$x + 5 - \underline{\quad} = 1 - \underline{\quad}$$
$$x = \underline{\quad}$$

(d)
$$-2x = 14$$
$$\frac{-2x}{\boxed{\quad}} = \frac{14}{\boxed{\quad}}$$
$$x = \underline{\quad}$$

(e) $-6x = -12$

(f) $\frac{1}{4}x = -3$

(g) $x - 12 = 8$

(h) $x + 7 = 13$

(i) $\frac{x}{10} = 11$

(j) $x - 9 = -3$

(k) $x - 12 = 13$

(l) $-3x = 27$

B. Solutions Which Involve More Than One Operation

You were able to solve each of the equations in the last exercise by using only one of the properties of equality. More complex equations require the application of several properties before the solution can be obtained.

EXAMPLE: Solve the equation $3x - 7 = 5$.

Solution

First, in order to undo the operation of subtracting 7, we must add 7 to both sides of the equation.

ADD 7 TO BOTH SIDES.

$$3x - 7 = 5$$
$$3x - 7 + 7 = 5 + 7$$
$$3x = 12$$

Then, in order to undo the operation of multiplying x by 3, we must divide both sides of the equation by 3.

DIVIDE BOTH SIDES BY 3.

$$\frac{3x}{3} = \frac{12}{3}$$

$$x = 4$$

Self-correcting Exercise #2

Answers to this exercise may be found on page 53 of this lesson.

1. Describe the operation that was performed at each step in the solutions.

(a) $4x - 9 = 3$

add 9 to each side.

$4x = 12$

____

$x = 3$

(b) $2x + 1 = -3$

____

$2x = -4$

____

$x = -2$

(c) $\frac{1}{2}x + 4 = 6$

____

$\frac{1}{2}x = 2$

____

$x = 4$

(d) $\frac{x}{3} - 1 = 7$

____

$\frac{x}{3} = 8$

____

$x = 24$

2. Solve the following equations.

(a) $2x + 1 = -3$

$2x + 1 - \_\_ = -3 - \_\_$

$2x = \_\_\_$

$\frac{2x}{\square} = \frac{\square}{\square}$

$x = \_\_$

(b) $\frac{x}{5} - 3 = 1$

$\frac{x}{5} - 3 + \_\_ = 1 + \_\_$

$\frac{x}{5} = \_\_$

$\_\_ \times \frac{x}{5} = \_\_ \times \_\_$

$x = \_\_$

(c) $-3x + 4 = -8$

(d) $\frac{1}{2}x + 3 = 7$

In some linear equations, you will find that an x-term occurs on both sides of the equation. In such cases, you must add or subtract the appropriate amount so that an x-term appears only on the left side.

EXAMPLE: Solve the equation $5x + 3 = 6x - 1$

Solution

Note that we have an x-term on each side of the equation. In order to remove the term 6x from the right side, we can subtract 6x from both sides of the equation.

$$5x + 3 - 6x = 6x - 1 - 6x$$

$(5x - 6x = -x)$ $(6x - 6x = 0)$

$$-x + 3 = -1$$

Subtract 3 from each side.

$$-x + 3 - 3 = -1 - 3$$

$$-x = -4$$

Multiply each side by -1. (We haven't solved for x yet since it still has a minus sign in front of it. To get rid of this minus sign, we multiply both sides of the equation by -1.)

$$-1 \times -x = -1 \times -4$$

$$x = 4$$

Self-correcting Exercise #3

Answers to this exercise may be found on page 54 of this lesson.

1. Describe the operation that was performed at each step in the solutions.

(a) $3x - 2 = 5x + 4$

____

$-2x - 2 = 4$

____

$-2x = 6$

____

$x = -3$

b. $6x + 3 = 3x + 15$

____

$3x + 3 = 15$

____

$3x = 12$

____

$x = 4$

(c) $3x + 4 = 2 - x$

$4x + 4 = 2$

$4x = -2$

$x = \frac{-1}{2}$

d. $5x - 2 = 6x - 8$

$-x - 2 = -8$

$-x = -6$

$x = 6$

2. Solve these equations.

(a) $2x - 10 = 15 + 3x$

b. $2x + 4 = 16 - 4x$

Some linear equations contain a number of x-terms and a number of constant terms (real numbers) on each side of the equation. When solving such equations, you must first combine any like terms which appear on one side of the equation. Then, the principles of equality can be applied.

EXAMPLE: Solve the equation $2(x - 3) + 4x = -3(x + 5) - 9$

Solution

$$2(x - 3) + 4x = -3(x + 5) - 9$$

First, simplify the terms $2(x - 3)$ and $-3(x + 5)$ by applying the distributive property.

$$2x - 6 + 4x = -3x - 15 - 9$$

We can simplify the left side by combining $2x$ and $4x$ to obtain $6x$. We can simplify the right side by combining -15 and -9 to obtain -24.

$$6x - 6 = -3x - 24$$

Add 3x to both sides.

$$6x - 6 + 3x = -3x - 24 + 3x$$

$$9x - 6 = -24$$

Add 6 to both sides.

$$9x - 6 + 6 = -24 + 6$$

$$9x = -18$$

Divide each side by 9.

$$\frac{9x}{9} = \frac{-18}{9}$$

$$\underline{\underline{x = -2}}$$

Self-correcting Exercise #4

Answers to this exercise may be found on page 54 of this lesson.

1. Describe the operation that was performed at each step of the following solutions.

(a) $4(x + 3) - 4 = 2(x - 3)$

____

$4x + 12 - 4 = 2x - 6$

Collect like terms.

$4x + 8 = 2x - 6$

____

$2x + 8 = -6$

____

$2x = -14$

____

$x = -7$

(b) $x + 3 - 5x = 8 - x - 9$

____

$-4x + 3 = -x - 1$

____

$-3x + 3 = -1$

____

$-3x = -4$

____

$x = \frac{4}{3}$

2. Solve each equation.

(a) $7x + 9 + 3x = 10 + 6x + 19$

(b) $-4(x + 2) = -3(x - 5)$

Some linear equations contain terms which are fractional in nature. When solving such equations, it is often convenient to clear the equation of fractions in the first step. This can be accomplished by multiplying each side of the equation by the L.C.D. of all the fractions involved.

EXAMPLE: Solve the equation $\dfrac{x + 4}{3} - \dfrac{x - 5}{4} = \dfrac{x}{6}$.

<u>Solution</u>

Note that the L.C.D. of the fractions is 12. (This means that 12 is the smallest number into which the denominators 3, 4, and 6 will divide.)

In order to clear the equation of fractions, multiply both sides by 12. (In other words, multiply <u>every</u> term by 12.)

$$\overset{4}{\cancel{12}}\left(\frac{x + 4}{\underset{1}{\cancel{3}}}\right) - \overset{3}{\cancel{12}}\left(\frac{x - 5}{\underset{1}{\cancel{4}}}\right) = \overset{2}{\cancel{12}}\left(\frac{x}{\underset{1}{\cancel{6}}}\right)$$

$$4(x + 4) - 3(x - 5) = 2x$$

Apply the distributive property on the left side of the equation.

NOTE SIGNS HERE.

$$4x + 16 - 3x + 15 = 2x$$

$$x + 31 = 2x$$

$$-x + 31 = 0$$

$$-x = -31$$

$$\underline{\underline{x = 31}}$$

<u>Self-correcting Exercise #5</u>

Answers to this exercise may be found on page 55 of this lesson.

1. Describe the operation that was performed at each step of the following solutions.

(a) $\frac{1}{2}x - \frac{2}{3} = \frac{x}{6}$

____

$3x - 4 = x$

____

$2x - 4 = 0$

____

$2x = 4$

____

$x = 2$

(b) $\frac{x+3}{12} + 5 = \frac{x-2}{8}$

____

$2(x+3) + 120 = 3(x - 2)$

____

$2x + 6 + 120 = 3x - 6$

____

$2x + 126 = 3x - 6$

____

$-x + 126 = -6$

____

$-x = -132$

____

$x = 132$

2. Solve each equation.

(a) $\frac{x}{3} - \frac{1}{2} = \frac{5x}{6}$

(b) $\frac{2x - 1}{7} = \frac{x + 4}{5}$

C. Checking your Solution to an Equation

Since it is possible that you may make a calculation error when solving an equation, you should check your answer by substituting it in both sides of the original equation. If the left and right members of the equation are indeed equal when the value is substituted, your answer should be correct.

Review the example on pages 22 and 23 of this lesson. Here, we solved the equation $2(x - 3) + 4x = -3(x + 5) - 9$ and found the root to be -2. We can check this root by substituting -2 for x in both sides of the original equation.

i.e. if $x = -2$,

$$\begin{array}{r|l}
2(x - 3) + 4x \overset{?}{=} & -3(x + 5) - 9 \\
2(-2 - 3) + 4(-2) & -3(-2 + 5) - 9 \\
2(-5) + (-8) & -3(3) - 9 \\
-10 + (-8) & -9 - 9 \\
-18 & -18
\end{array}$$

EQUAL

The root is correct since the equation is satisfied when $x = -2$.

EXERCISE - Solving Equations

1. Describe the operation that was performed at each step in the solutions.

a. $\frac{2x}{3} = 1$

Multiply each side by 3.

$2x = 3$

____

$x = 1\frac{1}{2}$

b. $\frac{1}{3}x - 1 = 5$

____

$\frac{1}{3}x = 6$

____

$x = 18$

c. $-2x + 3 = 7$

$-2x = 4$

$x = -2$

d. $\frac{x}{4} - 2 = 1$

$\frac{x}{4} = 3$

$x = 12$

e. $20 - 5x = 40 - 4x$

Add 4x to each side.

$20 - x = 40$

$-x = 20$

$x = -20$

f. $3x + 6 = 5x - 8$

$-2x + 6 = -8$

$-2x = -14$

$x = 7$

g. $5(x + 6) = 3(2x + 11)$

$5x + 30 = 6x + 33$

$-x + 30 = 33$

$-x = 3$

$x = -3$

h. $\frac{x + 3}{10} - \frac{x}{5} = \frac{x - 3}{15}$

$3(x + 3) - 6x = 2(x - 3)$

$3x + 9 - 6x = 2x - 6$

$-3x + 9 = 2x - 6$

$-5x + 9 = -6$

$-5x = -15$

$x = 3$

2. Solve the following equations and check your solutions.

(a) $2x - 5 = 7$
$2x = 12$
$x = 6$

Check
$2x - 5 \stackrel{?}{=} 7$
$2(6) - 5$
$12 - 5$
7

(b) $\frac{x}{8} - 3 = 2$

Check
$\frac{x}{8} - 3 \stackrel{?}{=} 2$

(c) $2x = 6 - 4x$

Check
$2x \stackrel{?}{=} 6 - 4x$

(d) $8x - 5 = 2x + 13$

Check
$8x - 5 \stackrel{?}{=} 2x + 13$

(e) $4 - 3x = 10 - 2x$

Check
$4 - 3x \stackrel{?}{=} 10 - 2x$

(f) $6m - 5 + 2m = 20 - m + 11$

Check
$6m - 5 + 2m \stackrel{?}{=} 20 - m + 11$

Check

(g) $\frac{5n}{3} = -60$

$\frac{5n}{3} \stackrel{?}{=} -60$

Check

(h) $7x - (2 + x) = 2(4x - 2)$

$7x - (2 + x) \stackrel{?}{=} 2(4x - 2)$

Check

(i) $\frac{x}{4} = \frac{x}{5} + \frac{7}{10}$

$\frac{x}{4} \stackrel{?}{=} \frac{x}{5} + \frac{7}{10}$

Check

(j) $\frac{x+1}{3} - \frac{x-1}{2} = \frac{x-1}{2} - \frac{3x-1}{6}$

$^{2}\cancel{6}\left(\frac{x+1}{\cancel{3}_{1}}\right) - {}^{3}\cancel{6}\left(\frac{x-1}{\cancel{2}_{1}}\right) = {}^{3}\cancel{6}\left(\frac{x-1}{\cancel{2}_{1}}\right) - {}^{1}\cancel{6}\left(\frac{3x-1}{\cancel{6}_{1}}\right)$

$2(x+1) - 3(x-1) = 3(x-1) - (3x-1)$

$=$

$\frac{x+1}{3} - \frac{x-1}{2} \stackrel{?}{=} \frac{x-1}{2} - \frac{3x-1}{6}$

Topic Five: Using Equations to Solve Problems

Mathematical phrases and mathematical equations are two completely different things.

1. A mathematical phrase involves number symbols, letter symbols, and operation symbols.
   EXAMPLES:
   $$3x + 5$$
   $$x - \frac{2}{3}y$$

2. A mathematical equation involves number symbols, letter symbols, operation symbols, and the relation symbol "=".
   EXAMPLES:
   $$x + 3 = 7$$
   $$2x - 5 = 3x + 12$$

If we wish to make a statement of equality regarding a number whose value is not known, we can write an equation using a variable to represent the unknown number. Remember that in an equation we write a statement about two things that are equal to each other.

Once we have written an equation to represent a certain problem situation, we can apply the four principles given in the box on page 14 of this lesson to solve the equation and find the value of the unknown.

Translate each statement below into an equation. (Let n represent the unknown number.) Then solve the equation in order to find the number.

1. Twice a certain number increased by 5 is equal to 13.

   Equation: $2n + 5 = 13$

   Solution: $2n = 8$

   $n = 4$

2. Five times a number decreased by 5 is equal to 20.

   Equation:

   Solution:

3. A certain number plus three times this number is 28.

   Equation:

   Solution:

4. One-half a number increased by 7 will equal 11.

   Equation:

   Solution:

5. The number 8 decreased by 3 times a certain number is equal to 2.

   Equation:

   Solution:

6. If a number is divided by 2, and 3 is added to the quotient, the result is 9.

   Equation:

   Solution:

Equations can be used to solve more complex problems. Use the following procedure.

Step 1. Use a variable to stand for the unknown quantity. (If there is more than one unknown quantity, you usually let the variable stand for the smallest.)

Step 2. Express any other unknown quantity using the same variable.

Step 3. From the data given in the problem, set up an equation.

Step 4. Solve the equation.

Step 5. Make a final statement.

Step 6. Check your answer back in the problem (Mental Step).

Solve the following problems by going through the required steps.

1. Helen's age is six years less than twice Mary's age. The sum of their ages is 30 years. What is the age of each girl?

Let x by Mary's age. STEP 1
Then Helen's age is (2x - 6) years. STEP 2

Equation: $x + (2x - 6) = 30$ STEP 3

$3x - 6 = 30$

$3x = 36$

$x = 12$ ← Mary's age

Thus $2x - 6 = 18$ ← Helen's age

STEP 4

Statement: Mary is 12 years old and Helen is 18 years old. STEP 5

Do these answers check with the wording of the problem? _____ STEP 6

2. The larger of two numbers is three times the smaller. If their sum is 84, find each number.

Let x be the smaller number.
Then the larger number is 3x.

Equation: $x + \_\_\_ = 84$

$\_\_\_ = 84$

$x = \_\_\_$ ← smaller number

Thus, $3x = \_\_\_$ ← larger number

Statement: ______________________________

3. The number of girls in Mr. Green's biology class is 7 less than twice the number of boys. The class enrollment is 29. How many girls and how many boys are in the class?

Let n be the number of boys.
Then (__n - __) is the number of girls.

Equation: n + (_____) = ___

Statement: ______________________________

4. During a period of three weeks, Mike Adams worked part time. He earned $7 more the first week than the second week. The third week he earned twice as much as the second week. He earned $51 for the three week period. How much did he earn each week?

Let $y be the amount Mike earned the <u>second</u> week.
Then $(y + __) is the amount he earned the first week
and $___y is the amount he earned the third week.

Equation: y + (_____) + ___ = 51

_y + ___ = 51

__y = ___

y = ___ ← second week

Thus, y + ___ = ___ ← first week

_y = ___ ← third week

Statement: ______________________________

5. The length of a rectangle is seven cm greater than the width. If the perimeter of the rectangle is 54 cm, what are the dimensions of the rectangle?

Let the width of the rectangle be x cm.
Then the length is (_____) cm.

(2 × Width) + (2 × Length) = Perimeter of Rectangle

2x + 2(_____) = ___

2x + ___ + ___ = ___

__x + ___ = ___

__x = ___

x = ___ ← width

← length

Statement: ____

6. If five times a number is decreased by 8, the result is the same as when the number is increased by 8. What is the number?

Let the number be n.
Five times the number, decreased by 8, is (_n - __).

The number increased by 8 is __________.

Equation: __n - ___ = n + ___

Statement: ____

7. Don, Tim, and Jack pooled their money to start a small business. Don invested twice as much as Jack. Tim invested $16 more than Jack. If the total amount invested by the three boys is $96, how much did each boy invest?

Let the amount that Jack invested be $x.

Then Don invested $__ and Tim invested $_____.

Equation: ___ + ___ + _____ = ___

Statement: ____

8. If the second angle of a triangle has a measure 10° more than the first, and the third angle has a measure 20° more than the first, what is the measure of each angle?
(Remember: The sum of the measures of the three angles of any triangle is always 180°.)

Let x° be the measure of the first angle.

Then (_____)° is the measure of the second angle and

(_____)° is the measure of the third angle.

Equation:

Statement: ____

Topic Six: Linear Inequalities in One Variable

A. Inequalities

Recall the meaning of the following symbols of inequality.

$>$ means "is greater than."

$\geqq$ means "is greater than or equal to."

$<$ means "is less than."

$\leqq$ means "is less than or equal to."

AN INEQUALITY is a mathematical statement which uses one of the above symbols to link two numerical or variable expressions. The following are examples of inequalities.

(1) $2 + 3 < 6$

(2) $-5 + 2 > 0$

(3) $x + 1 > 3$

(4) $x \leqq y$

Some inequalities involve real numbers only and are either true or false. Inequalities (1) and (2) above are of this type.

(1) $2 + 3 < 6$ is a number inequality whose left side equals 5 and right side equals 6. Since the left side of the inequality is less than the right side, this is a true statement.

(2) $-5 + 2 > 0$ is a number inequality whose left side equals -3 and right side equals 0. Since the left side of the inequality is less than the right side, this is a false statement.

Give examples of three other number inequalities that are true.

$5-4<0+2$ ________, ________, ________

Give examples of three other number inequalities that are false.

________, ________, ________

Decide whether each of the following number inequalities is true or false.

$3 \times 4 < 4 \times 3$	$-2 \times 3 < 5$	$-2 > -1$
false	________	________

$3 \times 6 < 9 + 2$ $\qquad$ $5 + 7 > 15 - 5$ $\qquad$ $2 + 3 \leqq 7$

__________ __________ __________

While some inequalities involve real numbers only, others involve variables. If an inequality contains at least one variable, it is called a CONDITIONAL INEQUALITY. As it stands, a conditional inequality is neither true nor false. It becomes true or false only when specific values have been assigned to the variables involved. Inequalities (3) and (4) at the top of page 36 of this lesson are conditional inequalities.

(3) $x + 1 > 3$ is a conditional inequality involving the one variable x. If we assign the value 2 to x, we obtain the number inequality $2 + 1 > 3$ which is false. On the other hand, if we assign the value 3 to x, we obtain the number inequality $3 + 1 > 3$ which is true.

(4) $x \leqq y$ is a conditional inequality involving the two variables x and y. If we assign the value -5 to x and the value -3 to y, we obtain the number inequality $-5 \leqq -3$ which is true. On the other hand, if we assign the value 5 to x and the value 3 to y, we obtain the number inequality $5 \leqq 3$ which is false.

EXERCISE - Inequalities

1. Translate each statement below into a conditional inequality. Let n represent the unknown number.

(a) Three times an unknown number decreased by 5 is less than or equal to 50. $3n - 5 \leqq 50$

(b) When an unknown number is doubled and then increased by 1, the result is greater than 7. __________

(c) If a number is divided by 7 and the quotient is decreased by 5, the result is greater than or equal to 41. __________

(d) One-third of a number is less than or equal to 99. __________

(e) A certain number plus five times this number is greater than 60. __________

2. Classify each of the following inequalities as being true, false, or conditional.

(a) $7 - 3 < 4 + 2$ ______

(b) $-2 - 1 < -4$ ______

(c) $y + 7 > 3$ ______

(d) $m + n \geqq 12$ ______

(e) $2 \times -2 < 2 \times 2$ ______

(f) $6 + 5n > 3$ ______

3. For each conditional inequality below, replace the variables by the numbers indicated and decide whether the resulting number inequality is true or false.

(a) $x + y \leqq 3$ Replace x by 7 and y by -4. $7 + (-4) \leqq 3$ $3 \leqq 3$ (true)	(b) $3m + 5 > 6$ Replace m by 0.	(c) $2xy - 3 < -4$ Replace x by 2 and y by -1.
(d) $x^2 + 1 \leqq 0$ Replace x by -2.	(e) $x + 1 \geqq y + 3$ Replace x by 4 and y by 2.	(f) $3a + 2b > 11$ Replace a by 2 and b by 3.

B. The Solution Set of a Linear Inequality in One Variable

A LINEAR INEQUALITY IN ONE VARIABLE is a conditional inequality of degree one which involves only one variable. The following are examples of inequalities of this type:

THE VARIABLE x IS TO THE FIRST POWER.

$$x < 5$$
$$x + 1 \geqq 5$$
$$2x + 1 \leqq 8x - 3$$
$$\frac{y}{4} > 9$$

In dealing with linear inequalities in one variable, we are interested in finding any values from the domain of the variable which will make the inequality a true statement. This subset of the domain which contains values of the variable that make the inequality true is called the SOLUTION SET of the inequality. Each individual member of the solution set is called a ROOT or SOLUTION of the inequality. The solution set of an inequality can be graphed on the number line.

EXAMPLE: If the domain of x is $A = \{5, 6, 7\}$, find the solution set of the inequality $3x + 1 \geqq 19$. Graph this solution set on the number line.

Solution

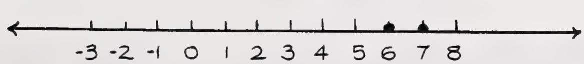
Substitute each value from the domain of x and see if the resulting number inequality is true or false.

If $x = 5$,
$$3(5) + 1 \geqq 19$$
$$15 + 1 \geqq 19$$
$$16 \geqq 19$$
(false)

If $x = 6$,
$$3(6) + 1 \geqq 19$$
$$18 + 1 \geqq 19$$
$$19 \geqq 19$$
(true)

If $x = 7$,
$$3(7) + 1 \geqq 19$$
$$21 + 1 \geqq 19$$
$$22 \geqq 19$$
(true)

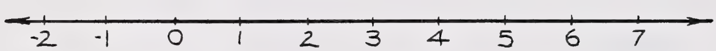
Since we obtain a true statement when $x = 6$ and when $x = 7$, we can say that both 6 and 7 are roots of the inequality $3x + 1 \geqq 19$ when $x \in \{5, 6, 7\}$. Thus, the solution set of the inequality for the given domain is $\{6, 7\}$. This solution set can be graphed as follows:



For the inequality $2x - 5 \leqq 1$, suppose that the domain of x is $\{0, 1, 2, 3, 4, 5\}$. Test each element from the domain in the inequality and decide if the resulting number inequality is true or false. Then state the solution set of the inequality and graph it on the number line.

If $x = 0$, $2(0) - 5 \leqq 1$ $0 - 5 \leqq 1$ $-5 \leqq 1$ (true)	If $x = 1$,	If $x = 2$,
If $x = 3$,	If $x = 4$,	If $x = 5$,

We obtain a true statement when x = ___, ___, ___, and ___. Thus, the solution set of the inequality is {___, ___, ___, ___} for the given domain. Graph this solution set on the number line provided below.



If the domain of the variable in a linear inequality contains an infinite number of elements, it is impossible to test all the values of the variable in the given inequality. In such cases, the solution set can often be determined by inspection.

EXAMPLE: If $x \in R$, find the solution set of the inequality $2x < 8$. Graph this solution set on the number line.

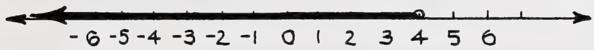
Solution

$$2x < 8$$

In this case, we are seeking a value of x which when multiplied by 2 will be <u>less</u> than 8. We know that if $x = 4$, the left member will equal 8 but will not be less than 8. But, if we choose any real value of x which is less than 4, the left member will always be less than 8 and the inequality will become a true statement.

Thus, the solution set of the inequality $2x < 8$ when $x \in R$ is $\{x|x < 4, x \in R\}$.

This solution set can be graphed as follows:



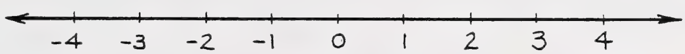
} Review graphs of this type on pages 23 and 24, lesson 6.

Solve each of the inequalities below by inspection. (In each case, assume that the domain of x is R.) Then graph the solution set.

1. $x + 3 > 5$

   This inequality is satisfied when $x > \underline{\quad}$ since it becomes a true statement whenever any real number greater than $\underline{\quad}$ is substituted for x. The solution set is $\{x|x > \underline{\quad}, x \underline{\quad} R\}$.

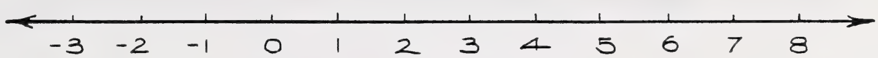
   Graph:



2. $x - 4 \leqq 3$

   This inequality is satisfied when $x \leqq \underline{\quad}$ since it becomes a true statement whenever any real number less than or equal to $\underline{\qquad}$ is substituted for x. The solution set is $\{x|\underline{\qquad}, x \underline{\quad} R\}$.

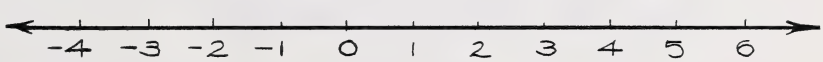
   Graph:



3. $\frac{x}{2} \geqq 1$

   This inequality is satisfied when $x \geqq \underline{\quad}$ since it becomes a true statement whenever any real number greater than or equal to $\underline{\qquad}$ is substituted for x. The solution set is $\{x|\underline{\qquad\qquad\qquad}\}$.

   Graph:



Topic Seven: Formal Method for Solving Linear Inequalities in One Variable

The solution set of a linear inequality in one variable cannot always be easily determined by inspection. We must devise a formal method for solving inequalities of this type. Our aim will be to write a series of equivalent inequalities (i.e. inequalities with the same solution set) until we arrive at one in which the variable appears alone in the left member and a real number appears alone in the right member.

In general, you will find that an equivalent inequality is obtained if:

1. The same real number is added to both sides.
2. The same real number is subtracted from both sides.
3. Each side is multiplied by the same <u>positive</u> real number.
4. Each side is divided by the same <u>positive</u> real number.

1. <u>Use the operation of addition to undo a subtraction.</u>

EXAMPLE: $x - 3 < 4$

In order to undo the operation of subtracting 3 from x, add 3 to both sides of the inequality.

$$x - 3 + 3 < 4 + 3$$

3 HAS BEEN ADDED TO BOTH SIDES.

$$\boxed{x < 7}$$

The solution set is $\{x|x < 7,\ x \in R\}$.

Solve the following inequalities by adding the same amount to each side.

$x - 5 \geqq -2$	$x - 1 < 9$	$x - 3 > -7$
$x - 5 + \underline{5} \geqq -2 + \underline{5}$	$x - 1 + \underline{\quad} < 9 + \underline{\quad}$	$x - 3 + \underline{\quad} > -7 + \underline{\quad}$
$x \geqq \underline{\quad}$	$x < \underline{\quad}$	$x > \underline{\quad}$

2. <u>Use the operation of subtraction to undo an addition.</u>

EXAMPLE: $x + 7 > 5$

In order to undo the operation of adding 7 to x, we must subtract 7 from <u>both</u> sides of the inequality.

$$x + 7 - 7 > 5 - 7$$

$$\boxed{x > -2}$$

7 IS SUBTRACTED FROM BOTH SIDES.

The solution set is $\{x|x>-2,\ x \in R\}$

Solve the following inequalities by subtracting the same amount from each side.

$x + 5 \geqq 12$	$x + 1 < -3$	$x + 6 > 2$
$x + 5 - \underline{5} \geqq 12 - \underline{5}$	$x + 1 - \_\_ < -3 - \_\_$	$x + 6 - \_\_ > 2 - \_\_$
$x \geqq \_\_\_$	$x < \_\_\_$	$x > \_\_\_$

3. Use the operation of multiplication to undo a division.

EXAMPLE: $\frac{x}{3} \leqq -5$

In order to undo the operation of dividing x by 3, we must multiply both sides of the inequality by 3.

$$\cancel{3} \times \frac{x}{\cancel{3}} \leqq 3 \times -5$$

EACH SIDE HAS BEEN MULTIPLIED BY 3.

$$\boxed{x \leqq -15}$$

The solution set is $\{x | x \leqq -15, \ x \in R\}$.

Solve the following inequalities by multiplying each side by the same positive real number.

$\frac{x}{6} > 3$	$\frac{1}{3}x < \frac{3}{4}$	$\frac{x}{2} \geqq -1$
$\underline{6} \times \frac{x}{6} > \underline{6} \times 3$	$\_\_ \times \frac{1}{3}x < \_\_ \times \frac{3}{4}$	$\_\_ \times \frac{x}{2} \geqq \_\_ \times -1$
$x > \_\_\_$	$x < \_\_\_$	$x \geqq \_\_\_$

4. Use the operation of division to undo a multiplication.

EXAMPLE: $5x < -20$

In order to undo the operation of multiplying x by 5, we must divide each side of the inequality by 5.

$$\frac{5x}{5} < \frac{-20}{5}$$

EACH SIDE HAS BEEN DIVIDED BY 5.

$$\boxed{x < -4}$$

The solution set is $\{x | x < -4, \ x \in R\}$

Solve the following inequalities by dividing each side by the same positive real number.

$4x \geqq 32$	$6x < 3$	$2x > -1$
$\frac{4x}{\boxed{4}} \geq \frac{32}{\boxed{4}}$	$\frac{6x}{\boxed{}} < \frac{3}{\boxed{}}$	$\frac{2x}{\boxed{}} > \frac{-1}{\boxed{}}$
$x \geqq$ ___	$x <$ ___	$x >$ ___

An equivalent inequality is NOT obtained if both sides of the inequality are multiplied or divided by the same negative real number. Study the following examples.

EXAMPLE 1: $3 < 5$

This is a true statement.
If we multiply both sides by -3, we obtain the inequality.

$$-9 < -15$$

This is a false statement. (-9 is greater than -15 since it lies to the right of -15 on the number line.)

In order to make this a true statement, we must change the direction of the arrow.

$$-9 > -15 \text{ (true)}$$

Thus, whenever we multiply both sides of an inequality by a negative real number, we must change the direction of the arrow.

EXAMPLE 2: $6 > -4$

This is a true statement.
If we divide both sides by -2, we obtain the inequality

$$-3 > 2$$

This is a false statement (-3 is less than 2 since it lies to the left of 2 on the number line.)

In order to make this a true statement, we must change the direction of the arrow.

$$-3 < 2 \text{ (true)}$$

Thus, whenever we divide both sides of an inequality by a negative real number, we must change the direction of the arrow.

> In general, an equivalent inequality is obtained if each side is multiplied or divided by the same <u>negative</u> real number and the <u>direction of the arrow is changed.</u>

EXAMPLE 1: Solve the inequality $-5x \geqq -30$.

<u>Solution</u>

$$-5x \geqq -30$$

In order to undo the operation of multiplying x by -5, we must divide each side of the inequality by -5. But when we divide by a negative number, we must change the direction of the arrow.

DIRECTION OF ARROW HAS BEEN CHANGED.

$$\frac{-5x}{-5} \leqq \frac{-30}{-5}$$

$$\boxed{x \leqq 6}$$

The solution set is $\{x | x \leqq 6,\ x \in R\}$

Solve the following inequalities by dividing each side by the same <u>negative</u> real number and <u>changing the direction of the arrow</u>.

$-7x < 14$	$-2x > 16$	$-4x \geqq -24$
$\frac{-7x}{\square} > \frac{14}{\square}$	$\frac{-2x}{\square} \square \frac{16}{\square}$	$\frac{-4x}{\square} \square \frac{-24}{\square}$
$x > -2$	______	______

EXAMPLE 2: Solve the inequality $\frac{x}{-3} < 7$.

<u>Solution</u>

$$\frac{x}{-3} < 7$$

In order to undo the operation of dividing x by -3, we must multiply both sides of the inequality by -3. But, when we multiply by a negative number, we must change the direction of the arrow.

DIRECTION OF ARROW HAS BEEN CHANGED.

$$-\overset{1}{\cancel{3}} \times \frac{x}{-\underset{1}{\cancel{3}}} > -3 \times 7$$

$$\boxed{x > -21}$$

The solution set is $\{x | x > -21,\ x \in R\}$

Solve the following inequalities by multiplying each side by the same <u>negative</u> real number and <u>changing the direction of the arrow</u>.

$-x > 5$	$\frac{-1}{2}x \leqq -3$	$\frac{x}{-9} \geqq 1$
$\underline{-1} \times -x < \underline{-1} \times 5$	$\boxed{-2} \times \frac{-1}{2}x \ \_\_ \ \Box \times -3$	$\Box \times \frac{x}{-9} \ \_\_ \ \Box \times 1$
$x < \_\_$	$x \ \_\_ \ \Box$	$x \ \_\_ \ \Box$

Thus, linear inequalities can be solved in a similar manner to linear equations except for one major difference:

> IF AN INEQUALITY IS MULTIPLIED OR DIVIDED BY THE SAME <u>NEGATIVE</u> REAL NUMBER, THE DIRECTION OF THE ARROW MUST BE CHANGED.

EXAMPLE: Find and graph the solution set of the inequality $6(-4 - y) \leqq 5(2y + 3) - 55$ where $y \in R$.

<u>Solution</u>

$$6(-4 - y) \leqq 5(2y + 3) - 55$$

First, write each member of the inequality in simplest form.

$$-24 - 6y \leqq 10y + 15 - 55$$
$$-24 - 6y \leqq 10y - 40$$

Subtract 10y from each side.

$$-24 - 16y \leqq -40$$

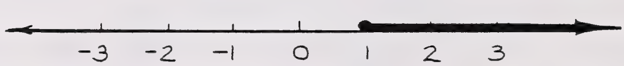
Add 24 to each side.

$$-16y \leqq -16$$

Divide each side by -16 <u>and change the direction of the arrow</u>.

$$y \geqq 1$$

The solution set is $\{y | y \geqq 1, y \in R\}$



Self-correcting Exercise #6

Answers may be found on page 55 of this lesson.

1. In each question, describe the operation that you would use to derive the second inequality from the first.

(a) $x + 12 < -5$ (1)
$x < -17$ (2)

____

____

(b) $\frac{x}{3} > -9$ (1)

$x > -27$ (2)

____

____

(c) $-2 + x \geqq -3$ (1)
$x \geqq -1$ (2)

____

____

(d) $-2x > 5$ (1)
$x < \frac{-5}{2}$ (2)

____

____

(e) $-x < 7$ (1)
$x > -7$ (2)

____

(f) $\frac{-1}{5}x \geqq -2$ (1)

$x \leqq 10$ (2)

____

2. Describe the operation that was performed at each step in the solutions.

(a) $2x + 5 < 1$

____

$2x < -4$

____

$x < -2$

(b) $\frac{-3}{4}x \geqq 6$

____

$-3x \geqq 24$

____

$x \leqq -8$

(c) $-4(x - 3) + 7 > 6 - 3(x + 1)$

$-4x + 12 + 7 > 6 - 3x - 3$

$-4x + 19 > 3 - 3x$

$-x + 19 > 3$

$-x > -16$

$x < 16$

(d) $-2x + 5 + 4x > 4 + 4x + 2$

$2x + 5 > 6 + 4x$

$-2x + 5 > 6$

$-2x > 1$

$x < \frac{-1}{2}$

3. Use the properties of inequality to find the solution set of each inequality in the set of real numbers. Then graph this solution set on the number line provided.

(a) $2x + 10 \leqq 0$

Solution set is

$\{x \mid$ ______________ $\}$.

Graph:

(b) $-4x - 5 < 7 - x$

Solution set is

$\{x \mid$ ______________ $\}$.

Graph:

EXERCISE - Linear Inequalities in One Variable

1. Fill in the blanks.

   (a) $x + 1 \geqq 3$ and $x \geqq 2$ are ____________ inequalities because they have the same solution set.

   (b) In order to find the solution set of the inequality $-2x < 3$, we must ____________ each side by ________ and change the ____________ of the arrow.

   (c) If $y \in \{7, 8, 9, 10\}$, the solution set of the inequality $y - 5 > 3$ is ____________.

   (d) The inequality $3x + 5 > 2x - 7$ is a ____________ inequality because it is of degree ____________.

   (e) If $x \in R$, the inequality $x + 1 > 7$ is a true statement for all real values of x which are ____________ than ________.

   (f) If $x \in R$, the solution set of the inequality $-8x < 24$ can be written in set-builder notation as ____________.

   (g) We must change the direction of the arrow whenever we multiply or divide both sides of an inequality by a ________ real number.

2. Describe the operation that was performed at each step of the following solutions.

(a) $-x + 5 > 2$

____

$-x > -3$

Multiply each side by -1. (change arrow)

$x < 3$

(b) $\frac{1}{2}x - 7 \leqq 11$

____

$\frac{1}{2}x \leqq 18$

____

$x \leqq 36$

(c) $\frac{2}{3}x - \frac{3}{4} \geqq \frac{1}{2}x$

____

$8x - 9 \geqq 6x$

____

$2x - 9 \geqq 0$

____

$2x \geqq 9$

____

$x \geqq \frac{9}{2}$

(d) $3x + 4 > 5x - 6$

____

$-2x + 4 > -6$

____

$-2x > -10$

____

$x < 5$

(e) $\frac{3x + 5}{-2} < 2$

____

$3x + 5 > -4$

____

$3x > -9$

____

$x > -3$

(f) $-3(x - 5) < 5x + 7$

____

$-3x + 15 < 5x + 7$

____

$-8x + 15 < 7$

____

$-8x < -8$

____

$x > 1$

3. Find and graph the solution set of each of the following inequalities.

(a) $2x + 4 \leqq 10 - x$

Solution set is $\{x |$ ______, $x \in R\}$.
Graph:

(b) $-12 - 3x < 2x + 8$

Solution set is ______________.
Graph:

(c) $\frac{x + 1}{3} > \frac{x - 1}{2}$

Solution set is

____

Graph:

(d) $2(x + 5) - x \leqq 3(x + 4)$

Solution set is

____

Graph:

Key to Self-correcting Exercises in Lesson 9

Exercise #1, page 16

1. (a) $-8 + 2 = \underline{-6}$
   (b) $-7 - 3 = \underline{-10}$
   (c) $6 - 8 = \underline{-2}$
   (d) $-13 - 4 = \underline{-17}$
   (e) $-5 + 4 = \underline{-1}$
   (f) $5 - 4 = \underline{1}$
   (g) $-5 - 4 = \underline{-9}$
   (h) $3 - 9 = \underline{-6}$
   (i) $1 - 2 = \underline{-1}$
   (j) $2 - 5 = \underline{-3}$
   (k) $3 \times 5 = \underline{15}$
   (l) $3 \times -5 = \underline{-15}$
   (m) $-2 \times -3 = \underline{6}$
   (n) $-6 \times 4 = \underline{-24}$
   (o) $-2 \times -2 = \underline{4}$
   (p) $\frac{6}{3} = \underline{2}$
   (q) $\frac{-9}{3} = \underline{-3}$
   (r) $\frac{-6}{3} = \underline{-2}$
   (s) $\frac{-6}{-3} = \underline{2}$
   (t) $\frac{12}{-4} = \underline{-3}$

2. (a) Divide each side by 2.
   (b) Subtract 3 from each side.
   (c) Multiply each side by 2.
   (d) Add 3 to each side.
   (e) Divide each side by -3.
   (f) Subtract 7 from each side.

3. (a) $x + 35 = 25$

   Subtract 35 from each side.

   $x + 35 - 35 = 25 - 35$

   $\boxed{x = -10}$

   (b) $x - 12 = -43$

   Add 12 to both sides.

   $x - 12 + 12 = -43 + 12$

   $\boxed{x = -31}$

   (c) $\frac{x}{9} = -7$

   Multiply both sides by 9.

   $\overset{1}{\cancel{9}}\left(\frac{x}{\underset{1}{\cancel{9}}}\right) = 9 \times -7$

   $\boxed{x = -63}$

   (d) $-8x = -48$

   Divide both sides by -8.

   $\frac{-\overset{1}{\cancel{8}}x}{-\underset{1}{\cancel{8}}} = \frac{-48}{-8}$

   $\boxed{x = 6}$

Exercise #2, page 20

1. (a) Add 9 to each side. Divide each side by 4.

   (b) Subtract 1 from each side. Divide each side by 2.

   (c) Subtract 4 from each side. Multiply each side by 2.

   (d) Add 1 to each side. Multiply each side by 3.

2. (a)

$$2x + 1 = -3$$

$$2x + 1 - 1 = -3 - 1$$ SUBTRACT 1 FROM EACH SIDE.

$$2x = -4$$

$$\frac{2x}{2} = \frac{-4}{2}$$ DIVIDE EACH SIDE BY 2.

$$\boxed{x = -2}$$

(b)

$$\frac{x}{5} - 3 = 1$$

$$\frac{x}{5} - 3 + 3 = 1 + 3$$ ADD 3 TO EACH SIDE.

$$\frac{x}{5} = 4$$

$$5 \times \frac{x}{5} = 5 \times 4$$ MULTIPLY EACH SIDE BY 5.

$$\boxed{x = 20}$$

(c)

$$-3x + 4 = -8$$ SUBTRACT 4 FROM EACH SIDE.

$$-3x + 4 - 4 = -8 - 4$$

$$-3x = -12$$

$$\frac{-3x}{-3} = \frac{-12}{-3}$$ DIVIDE EACH SIDE BY -3.

$$\boxed{x = 4}$$

(d)

$$\frac{1}{2}x + 3 = 7$$

$$\frac{1}{2}x + 3 - 3 = 7 - 3$$ SUBTRACT 3.

$$\frac{1}{2}x = 4$$

$$2 \times \frac{1}{2}x = 2 \times 4$$ MULTIPLY BY 2.

$$\boxed{x = 8}$$

Exercise #3, page 21

1. (a) Subtract 5x from each side.
Add 2 to each side.
Divide each side by -2.

(b) Subtract 3x from each side.
Subtract 3 from each side.
Divide each side by 3.

(c) Add x to each side.
Subtract 4 from each side.
Divide each side by 4.

(d) Subtract 6x from each side.
Add 2 to each side.
Multiply each side by -1.

2. (a) $2x - 10 = 15 + 3x$
Subtract 3x from each side.
$-x - 10 = 15$
Add 10 to each side.
$-x = 25$
Multiply each side by -1.
$x = -25$

(b) $2x + 4 = 16 - 4x$
Add 4x to each side.
$6x + 4 = 16$
Subtract 4 from each side.
$6x = 12$
Divide each side by 6.
$x = 2$

Exercise #4, page 23

1. (a) Apply distributive property.
Collect like terms.
Subtract 2x from each side.
Subtract 8 from each side.
Divide each side by 2.

(b) Collect like terms.
Add x to each side.
Subtract 3 from each side.
Divide each side by -3.

2. (a) $7x + 9 + 3x = 10 + 6x + 19$

Collect like terms.

$$10x + 9 = 6x + 29$$

Subtract 6x from each side.

$$4x + 9 = 29$$

Subtract 9 from each side.

$$4x = 20$$

Divide each side by 4.

$$\boxed{x = 5}$$

(b) $-4(x + 2) = -3(x - 5)$

Apply the distributive property.

$$-4x - 8 = -3x + 15$$

Add 3x to both sides.

$$-x - 8 = 15$$

Add 8 to both sides.

$$-x = 23$$

Multiply both sides by -1.

$$\boxed{x = -23}$$

Exercise #5, page 25

1. (a) Multiply every term by 6.
Subtract x from both sides.
Add 4 to both sides.
Divide both sides by 2.

(b) Multiply every term by 24.
Apply distributive property.
Collect like terms.
Subtract 3x from both sides.
Subtract 126 from both sides.
Multiply both sides by -1.

2. (a)

$$\frac{x}{3} - \frac{1}{2} = \frac{5x}{6}$$

Multiply every term by 6.

$$\overset{2}{\cancel{6}}\left(\frac{x}{\underset{1}{\cancel{3}}}\right) - \overset{3}{\cancel{6}}\left(\frac{1}{\underset{1}{\cancel{2}}}\right) = \overset{1}{\cancel{6}}\left(\frac{5x}{\underset{1}{\cancel{6}}}\right)$$

$$2x - 3 = 5x$$

Subtract 5x from both sides.

$$-3x - 3 = 0$$

Add 3 to both sides.

$$-3x = 3$$

Divide both sides by -3.

$$\boxed{x = -1}$$

(b)

$$\frac{2x - 1}{7} = \frac{x + 4}{5}$$

Multiply both sides by 35.

$$\overset{5}{\cancel{35}}\left(\frac{2x - 1}{\underset{1}{\cancel{7}}}\right) = \overset{7}{\cancel{35}}\left(\frac{x + 4}{\underset{1}{\cancel{5}}}\right)$$

$$5(2x - 1) = 7(x + 4)$$

Apply the distributive property.

$$10x - 5 = 7x + 28$$

Subtract 7x from both sides.

$$3x - 5 = 28$$

Add 5 to both sides.

$$3x = 33$$

Divide both sides by 3.

$$\boxed{x = 11}$$

Exercise #6, page 47

1. (a) Subtract 12 from both sides.

(b) Multiply each side by 3.

(c) Add 2 to each side.

(d) Divide each side by -2 (and change arrow).

(e) Multiply each side by -1 (and change arrow).

(f) Multiply each side by -5 (and change arrow).

2. (a) Subtract 5 from each side.
Divide each side by 2.

(b) Multiply each side by 4.
Divide each side by -3
(and change arrow).

(c) Apply the distributive property.
Collect like terms.
Add 3x to each side.
Subtract 19 from each side.
Multiply each side by -1
(and change arrow).

(d) Collect like terms.
Subtract 4x from each side.
Subtract 5 from each side.
Divide each by -2
(and change arrow).

3. (a) $2x + 10 \leqq 0$

Subtract 10 from each side.

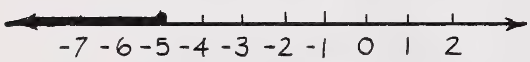
$2x \leqq -10$

Divide each side by 2.

$\boxed{x \leqq -5}$

Solution set is

$\{x | x \leqq -5,\ x \in R\}$



(b) $-4x - 5 < 7 - x$

Add x to both sides.

$-3x - 5 < 7$

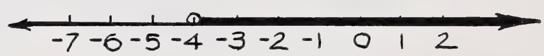
Add 5 to both sides.

$-3x < 12$

Divide both sides by -3 (and change direction of arrow).

$\boxed{x > -4}$

$\{x | x > -4,\ x \in R\}$



End of Lesson 9

Lesson 10

Geometry

Basic Algebra and Geometry

GEOMETRY

Caution: Don't attempt this lesson unless you have a ruler and a protractor.

Geometry has been studied by mankind as long as we have records. The need for measuring and laying out plots of land gave rise to the beginnings of geometry. For over 2,000 years, the science of geometry has provided mankind with the techniques for measuring and calculating distances, angles, areas, and volumes.

The geometry we study today is based on the work of a Greek mathematician, Euclid. This geometry deals with sets of points. In your previous study of mathematics, you have worked mainly with sets of numbers.

Topic One: Points, Lines, Planes

In any science, there are certain words basic to the vocabulary of that science that must generally be accepted as being undefined. For example, in physics, the word "electron" is difficult to define. In chemistry, the word "atom" is basic to the vocabulary but difficult to define precisely. In mathematics you have come in contact with the word "set" which may be only vaguely defined.

In geometry, we must use the words "point", "line", and "plane" without making any formal definition of them. Although we cannot define these terms precisely, we can give illustrations of them and gain some insight into the concepts they represent.

A. Point

A point may be described as an exact location in space. We can represent a point by making a dot on a piece of paper. Points are named by capital letters.

EXAMPLE: .A This is "point A".

In the box provided on the left, use dots to represent three points. Use three different capital letters to name these points.

Remember that a point has neither length, width, nor thickness. It has position only.

B. <u>Line</u>

Like a point, a line has neither width nor thickness. It differs from a point because it has infinite length.

A <u>line</u> is often described as an infinite set of points. Between any two points on a line there is another point. Also, there are no gaps between the points on a line. A line has no endpoints, but continues indefinitely in two directions. When we talk about "lines", we assume that they are "straight lines".

We may represent a line by drawing a stroke with a ruler and placing arrowheads at both ends of it. The arrowheads indicate that the line has infinite length. A line is named by referring to any two points that lie in it. <u>A double-headed arrow is then placed above these two capital letters that are used to name the line</u>.

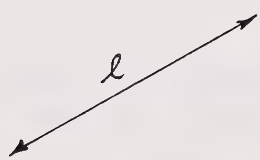
REMEMBER THE ARROW!

EXAMPLE:

This is "line AB" which can be written $\overleftrightarrow{AB}$.
Since the order in which we name the two points does not matter, this line could also be called $\overleftrightarrow{BA}$ (which is read, "line BA").

A line may also be named by associating a small letter (such as ℓ or m) with it.

EXAMPLE:



This is "line ℓ".

In the box provided on the left:

1. Represent two points and name them with capital letters.
2. Draw a line which passes through these points.

Name this line in two different ways.

________, ________

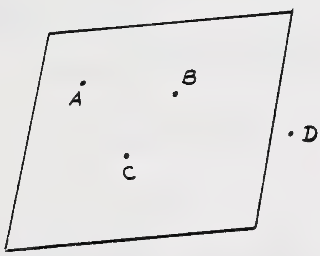
C. Plane

A plane is a set of points that makes up a flat surface extending in all directions without limit. To get the idea of a plane, think of a shadow on a flat surface. If the shadow extended indefinitely in all directions, it could represent a plane. A plane has infinite length and width, but no thickness.

We may represent a plane by drawing a four-sided figure on a piece of paper. Since three points that are not all in a straight line determine a plane, a plane may be named by referring to three points in it.

(Don't give more than 3.)

EXAMPLE:



This is "plane ABC".

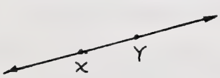
Since point D lies outside the parallelogram, it is not part of the plane.

In the space provided on the left, represent a plane and four different points in the plane. Use capital letters to name the points.

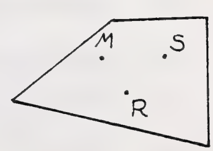
Name this plane in two different ways: plane ________, plane ________

EXERCISE - Points, Lines, Planes

1. Name each figure.

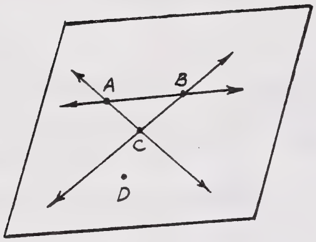
(a) 

(b) 

(c) 

________ ________ ________

2. Study the diagram and then answer the questions.
(The 4-sided figure represents a plane.)



(a) Name the plane. plane ________

(b) Does point D lie in the plane? ____

(c) Name three different lines in the plane.

$\overleftrightarrow{AB}$, ________, ________

(d) Name two lines that point B belongs to. ________, ________

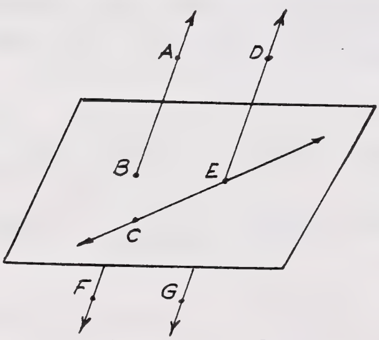
(e) Name two lines that point C belongs to. ________, ________

3.

Name this line in six different ways. (Remember that any two points can be used to name the line and these points can be listed in either order.)

$\overleftrightarrow{RQ}$, $\overleftrightarrow{QR}$, ________, ________, ________, ________

4.



The 4-sided figure represents a plane.

(a) Name the given plane. ________

(b) At what point does $\overleftrightarrow{DG}$ cut the plane? E

(c) At what point does $\overleftrightarrow{AF}$ cut the plane? ________

(d) Does $\overleftrightarrow{AF}$ lie in the given plane? no

(e) Does $\overleftrightarrow{CE}$ lie in the given plane? ________

(f) Name two points not in the given plane. A, ________

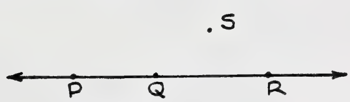
Topic Two: Subsets of a Line

In Lesson 1 of this course, you learned that one set is a subset of another set if all of its elements are contained in the larger set. For example, if A = {3, 4, 5} and B = {2, 3, 4, 5, 9}, set A is a subset of set B because every element of set A also belongs to set B.

We can also use the word "subset" when we are talking about sets of points. One set of points is a subset of another set if all of its points are contained in the larger set.

It is easy to see that any given point in a line is a subset of that line. Sets of points that are subsets of the same line are called COLLINEAR POINTS.

EXAMPLE:



In this case, point P is a subset of $\overleftrightarrow{PR}$.

Is point R a subset of $\overleftrightarrow{PR}$? ________

Is point S a subset of $\overleftrightarrow{PR}$? ________

Since points P, Q, and R lie in the same straight line, they are called collinear points.

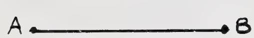
Line segments and rays are other important subsets of a line.

A. Line Segment

A line segment is a set of points in a line consisting of any two distinct points and all the points between them.

We may represent a segment by drawing a line of definite length on a piece of paper. A segment is named by referring to its two endpoints (in either order). A bar is then placed above these two capital letters that are used to name the segment. REMEMBER THE BAR!

EXAMPLE:



Make $\overline{AB}$ 3 cm long

This is "segment AB" which can be written $\overline{AB}$. (It could also be called "segment BA" and written $\overline{BA}$.)

Every segment has a definite length that can be measured with a ruler. We use the symbol d(A, B) to mean the "measure of segment AB." Since $\overline{AB}$ above has a measure of 3 cm, we can write d(A, B) = 3 cm.

Name this segment. ________

Name its endpoints. ___, ___

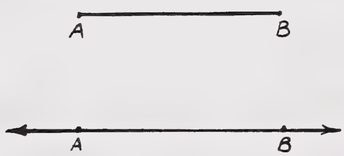
Measure the length of this segment to one decimal place. d(__) = __ cm

Any point in a segment that lies half-way between the endpoints is called the MIDPOINT of that segment. Use a ruler to locate the midpoint Q of $\overline{PR}$ below.

State the following measures.

d(P, Q) = ___ cm, d(Q, R) = ___ cm, d(P, R) = ___ cm

Every segment is part of a line, even though the line may not be shown.
EXAMPLE:



$\overline{AB}$ is a subset of $\overleftrightarrow{AB}$.
In other words all the points that lie in $\overline{AB}$ also lie in $\overleftrightarrow{AB}$.

B. <u>Ray</u>

A <u>ray</u> is a set of points in a line consisting of a given point and all the points on one side of it.

We may represent a ray by drawing a line and placing an arrowhead at <u>one</u> end of the line. The arrowhead indicates that the ray extends indefinitely in one direction. <u>A ray is named by stating the endpoint first and then giving one other point on the ray. An arrow pointing to the right is then placed above these two letters that are used to name the ray.</u> IMPORTANT!
EXAMPLE:

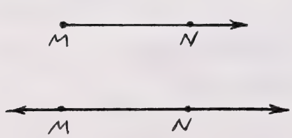
This is "<u>ray</u> AB" which can be written $\overrightarrow{AB}$. (Remember that the endpoint must be given first.)

Although the endpoint must always be given first when naming a ray, any other point which lies on the ray can be named as the second point.
EXAMPLE:

One name for this ray is $\overrightarrow{RS}$.
It could also be named $\overrightarrow{RT}$ or ________.

All three names refer to the <u>same</u> ray.

Every ray is part of a given line even though the line may not be shown.



$\overrightarrow{MN}$ is a subset of $\overleftrightarrow{MN}$.
In other words, all the points that lie in $\overrightarrow{MN}$ also lie in $\overleftrightarrow{MN}$.

In the space provided on the left:

1. Represent a line that passes through three points F, G, and H.

2. Name the line. __________

3. Name three different line segments that are subsets of the line.

$\overline{FG}$ ________, ________, ________

(Remember that $\overline{GF}$ is the same segment as $\overline{FG}$.)

4. Name three different rays that are subsets of the line.

($\overrightarrow{FG}$ & $\overrightarrow{FH}$ are the same ray, because both have the same endpoint and both occupy the same portion of the line.)

$\overrightarrow{FG}$ ________, ________, ________

EXERCISE - Subsets of a Line

1. Fill in the blanks.

(a) A __________ is a well-defined collection of objects. In arithmetic, we deal with sets of numbers. In geometry, we deal with sets of __________.

(b) When each member of one set is a member of another set, we say that the first set is a __________ of the other. Points, segments, and rays are __________ of a line.

(c) A __________ is a geometric figure that has position but no dimension.

(d) A line has infinite __________, but no width nor ____________.

(e) A __________ is part of a line that consists of two endpoints and all the points between.

(f) A __________ extends indefinitely in one direction.

(g) A line is named by referring to any two ____________ in the line and placing a doubled-headed __________ over these two capital letters.

(h) A segment has definite __________, but no width nor thickness.

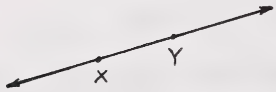
(i) The measure of segment RS is represented by the symbol _______.

(j) Points are ________________ if they are elements of the same line.

(k) P is called the __________________ of $\overrightarrow{PA}$.

2. In each question, name the line. Then name <u>one</u> segment and <u>two</u> rays that are subsets of the line.

(a)



Name of line ________

Name of segment ________

Names of rays $\overrightarrow{YX}$, ______

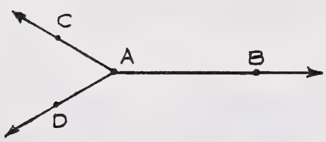
(b)

Name of line ________

Name of segment ________

Names of rays ______, ______

3.



(a) Name as many rays as you can. $\overrightarrow{AC}$, ____________

(b) Name as many segments as you can. $\overline{AB}$, ____________

(c) Are there any lines in this diagram? ________

4. Draw a diagram to represent each of the following geometric figures.

(a) $\overline{CD}$

(b) point W

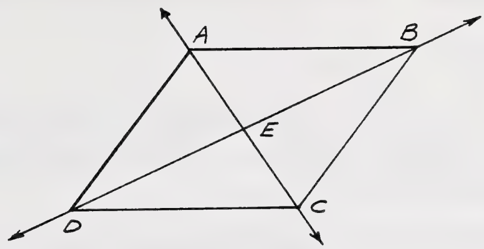
(c) $\overrightarrow{TS}$

(d) plane MNS

(e) $\overleftrightarrow{VW}$

(f) segment GH

5.



(a) Name 5 points. A, ____, ____, ____, ____

(b) Name 2 lines. $\overleftrightarrow{AC}$, ____

(c) Name 3 different segments that are subsets of $\overleftrightarrow{AC}$.

____, ____, ____

(d) Name 4 different rays that are subsets of $\overleftrightarrow{AC}$.

____, $\overrightarrow{AC}$, ____, ____

(Caution: Don't name the same ray twice. For example, note that $\overrightarrow{AC}$ and $\overrightarrow{AE}$ refer to the same ray.)

(e) Name 4 rays that have E as an endpoint.

____, ____, ____, ____

(f) Name 2 segments that have B as an endpoint. ____, ____

(g) What point is a subset of both $\overrightarrow{EA}$ and $\overrightarrow{EC}$? ____

(h) Since point E divides $\overline{AC}$ and $\overline{BD}$ in half, it is the ________ of both segments.

6. Draw $\overline{MN}$ 5 cm long. Locate point P on $\overline{MN}$ so that P is 2 cm from M. Using P as an endpoint, draw $\overrightarrow{PR}$ above $\overline{MN}$.

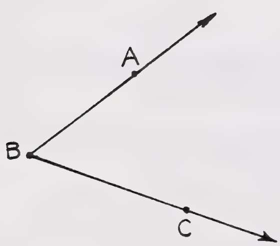
Topic Three: Angles

A. What is an Angle?

Although the word "angle" is part of our everyday vocabulary, we must remember that an angle is a mathematical idea and as such has a very precise definition.

> An angle is a set of points formed by two rays having the same endpoint.

We may represent an angle by drawing two rays that begin at the same point and go off indefinitely in two directions. The arrowhead at the end of each ray indicates that it extends indefinitely in one direction. The common endpoint of the rays is called the vertex of the angle.
EXAMPLE:



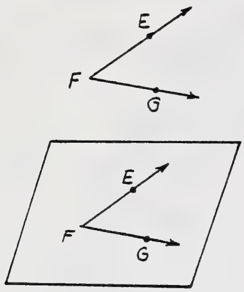
This figure represents an angle. The angle is composed of two rays, $\overrightarrow{BA}$ and $\overrightarrow{BC}$, that have the common endpoint, B. Point B is called the vertex of the angle.

In the space provided on the left, draw an angle composed of $\overrightarrow{ZX}$ and $\overrightarrow{ZY}$.

What is the vertex of this angle?

____

Every angle is part of a plane, even though the plane may not be shown.
EXAMPLE:



The angle which is composed of $\overrightarrow{FE}$ and $\overrightarrow{FG}$ is a subset of plane EFG.

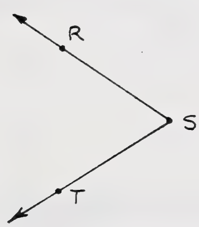
In other words, all the points which lie in the angle also lie in the plane.

B. <u>Naming Angles</u>

When writing about angles, we often use the symbol "∠" as a short form for the word "angle". Angles can be named either by:

1. Giving the vertex
(or) 2. Referring to three points in the angle - a point on one ray, the vertex, and a point on the other ray. (The middle letter given must always be the vertex.)

EXAMPLE:



MIDDLE LETTER IS THE VERTEX

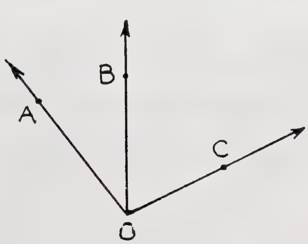
We can call this angle ∠S or ∠RST.

Name the two rays that make up this angle. _____, _____

What is the common endpoint of these rays? _____. This common endpoint is called the __________ of the angle.

Angles <u>must</u> be named by referring to three points if there are two or more angles having the same vertex. Confusion arises if only the vertex is given.

EXAMPLE:



In this case, ∠O does not name one specific angle. Each angle must be named by referring to three points. The three angles shown in this diagram are

∠AOC, ∠________, and ∠________.

Self-correcting Exercise #1

Answers to this exercise may be found on page 50 of this lesson.

1. Name each angle by giving the vertex only. Then name it by referring to 3 points.

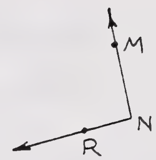
(a)

_____, _____

(b)

_____, _____

(c)

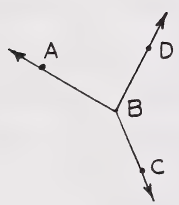


_____, _____

2. Name the two rays that make up each angle in #1.

(a) _____, _____ (b) _____, _____ (c) _____, _____

3.



Name three different angles.

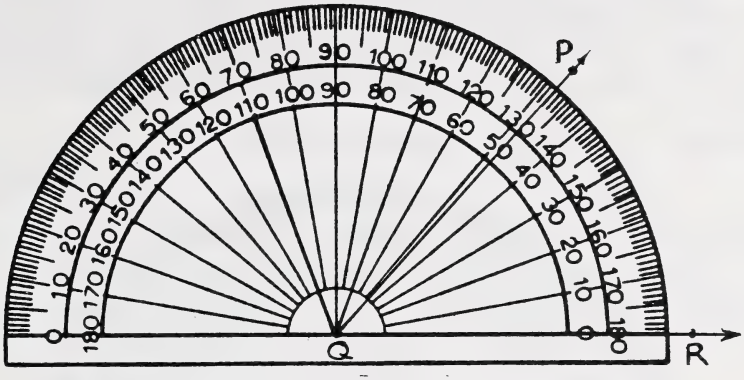
_______, _______, _______

C. Measuring Angles

The number of degrees in an angle is called the measure of the angle. The symbol $m\angle ABC$ represents the "measure of angle ABC." We use a protractor to find the measure of an angle. There are 180 equally spaced divisions marked around the edge of a protractor. Each of these divisions represents a measure of one degree. Protractors have two scales so that angles may be measured either clockwise or counterclockwise.

When an angle opens to the right, we usually use the counterclockwise (inner) scale on the protractor to find the measure. We place the center of the protractor at the vertex of the angle so that the zero mark on the inner scale falls on the lower ray of the angle. Then, on the inner scale we read the number through which the other ray passes.

EXAMPLE: Find the measure of ∠PQR.



Solution

1. Since ∠PQR opens to the right, we place the center of the protractor at Q and make sure that the zero mark on the inner scale lies on $\overrightarrow{QR}$.

2. Now, we count up from zero until we come to the mark through which $\overrightarrow{QP}$ passes. In this case, we can count 0, 10, 20, 30, 40, 45, 46, 47, 48.

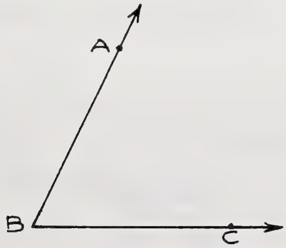
3. The measure of this angle is 48° (which is read, "forty-eight degrees").

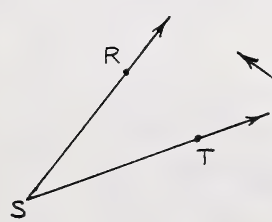
i.e. $m\angle ABC = 48°$

Self-correcting Exercise #2

Answers to this exercise may be found on page 50 of this lesson.

Use the inner scale of your protractor to measure each of the following angles (i.e. place the zero mark on the inner scale along the lower ray of the angle). Extend the rays of the angles if this helps you read the values more accurately.

1. 

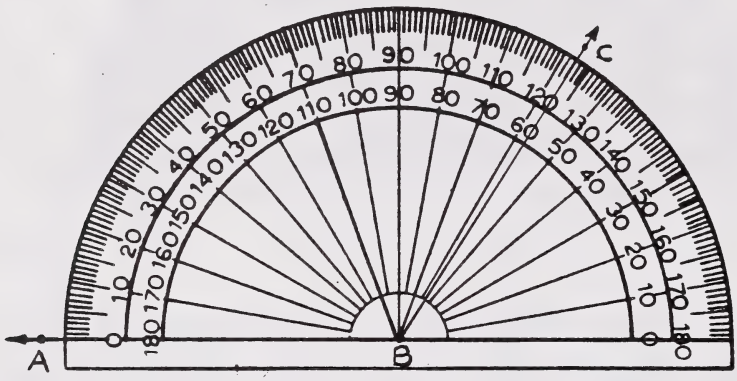
2. 

3. 

$m\angle ABC$ = ________ $m\angle RST$ = ________ $m\angle XYZ$ = ________

When an angle opens to the left, we usually use the clockwise (outer) scale on the protractor to find its measure. We place the centre of the protractor at the vertex of the angle so that the zero mark on the <u>outer</u> scale falls on the lower ray of the angle. Then, on the <u>outer</u> scale we read the number through which the other ray passes.

EXAMPLE: Find the measure of $\angle ABC$.



<u>Solution</u>

1. Since $\angle ABC$ opens to the left, we place the centre of the protractor at B and make sure that the zero mark on the <u>outer</u> scale lies on $\overrightarrow{BA}$.
2. Now, we count up from zero until we come to the mark through which $\overrightarrow{BC}$ passes. In this case, we can count 0, 10, 20, . . ., 80, 90, 100, 110, 120, 121, 122.
3. The measure of this angle is 122°.

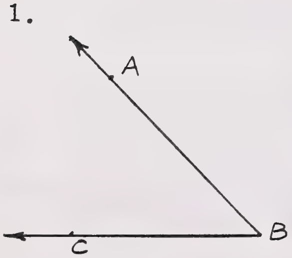
$$\text{ie. } m\angle ABC = 122°$$

<u>Self-correcting Exercise #3</u>

Answers to this exercise may be found on page 51 of this lesson.

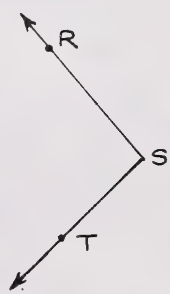
Use the <u>outer</u> scale of your protractor to measure each of the following angles (ie. place the zero mark on the outer scale along the lower ray of the angle).

1.



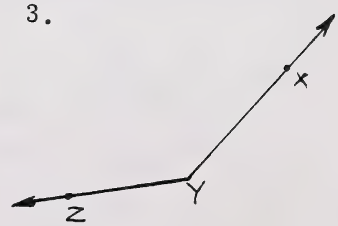
$m\angle ABC$ = ________

2.



$m\angle RST$ = ________

3.



$m\angle XYZ$ = ________

> When measuring angles, first place the protractor so that zero falls on one of the rays of the angle. Then, use the same scale that this zero mark belongs to in finding the number of degrees.

D. Drawing Angles

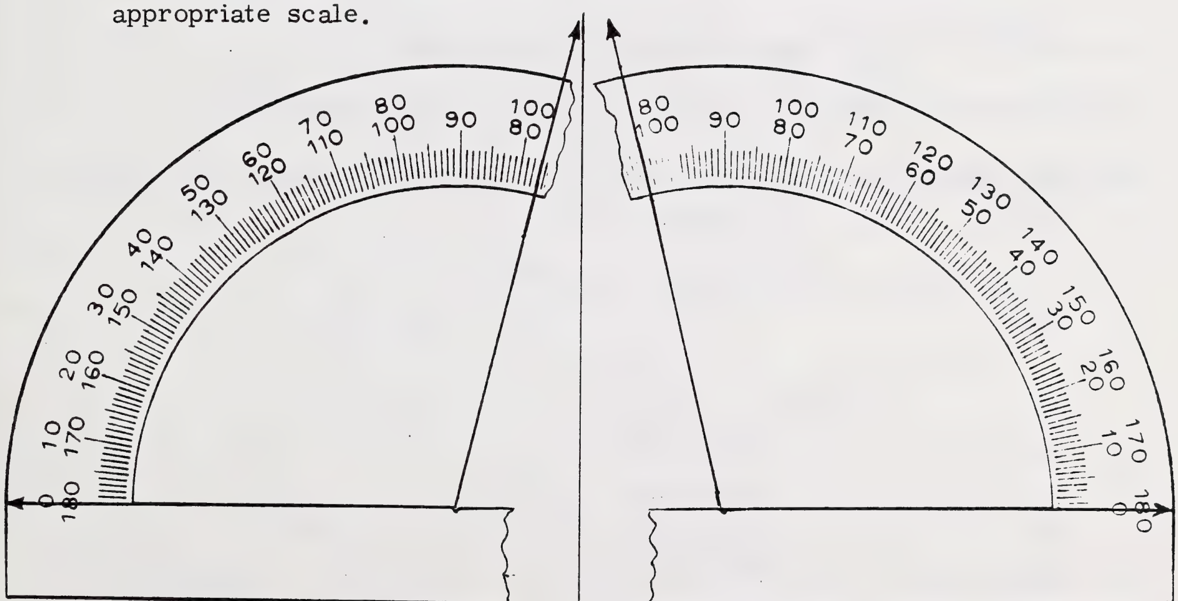
A protractor can also be used to draw an angle of a given size. We first draw a ray and then place the centre of the protractor at the endpoint of this ray. We look to see which zero mark on the protractor lies along the ray and use this scale to mark off the required number of degrees. We can then draw in the other ray.

EXAMPLE: Draw an angle whose measure is 103°.

Solution

We can draw this angle either opening to the left or to the right. First, draw one ray.

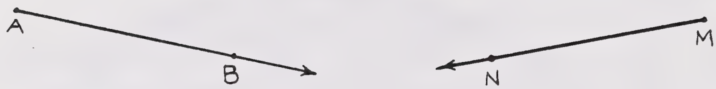
Then, place the protractor at the end of this ray and use the appropriate scale.



In this case, the zero mark on the outer scale lies along the ray. Thus, we must mark off 103° using the outer scale.

In this case, the zero mark on the inner scale lies along the ray. Thus, we must mark off 103° using the inner scale.

In the space provided below, draw an angle of 75° using $\overrightarrow{AB}$ and an angle of 125° using $\overrightarrow{MN}$. (Make vertices of angles at A and M.)



E. Classifying Angles

Angles may be classified according to their measures.

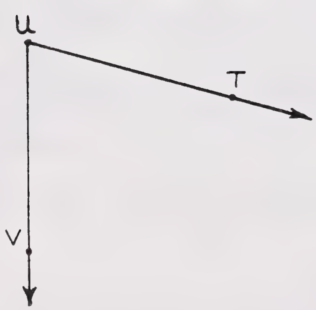
1. An angle with a measure of 90° is called a right angle.
2. An angle with a measure between 0° and 90° is called an acute angle.
3. An angle with a measure between 90° and 180° is called an obtuse angle.

Self-correcting Exercise #4

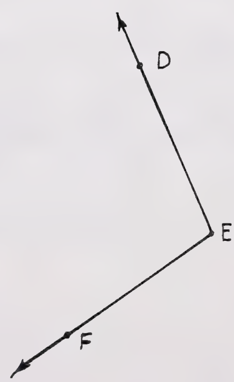
Answers to this exercise may be found on page 51 of this lesson.

State the measure of each angle and then classify it as being right, acute, or obtuse.

1.



2.



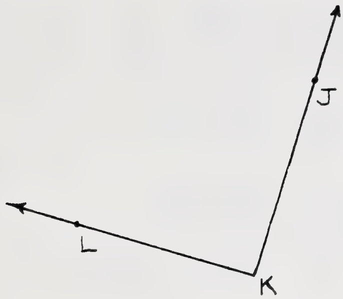
Measure: $m\angle$ TUV = _____ °

Classification: ____________

Measure: $m\angle$ _____ = _____ °

Classification: ____________

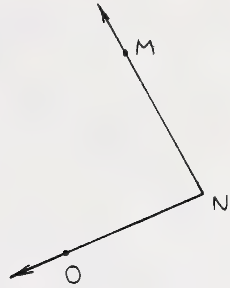
3.



Measure: $m\angle$ _____ = _____°

Classification: ____________

4.



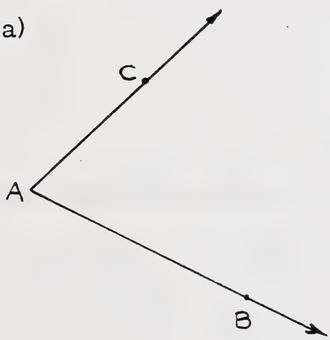
Measure: $m\angle$ _____ = _____°

Classification: ____________

EXERCISE - Angles

1. In each exercise, name the angle; name the rays of the angle; state the vertex; find the measure of the angle; and classify the angle as being right, acute or obtuse.

(a)



(i) Name $\angle CAB$

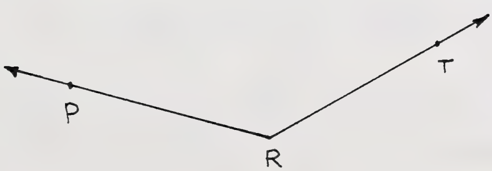
(ii) Rays $\overrightarrow{AC}$, _____

(iii) Vertex ____________

(iv) Measure ____________

(v) Classification ____________

(b)



(i) Name ____________

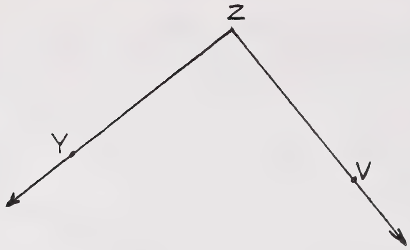
(ii) Rays _____ , _____

(iii) Vertex ____________

(iv) Measure ____________

(v) Classification ____________

(c)



(i) Name ____________

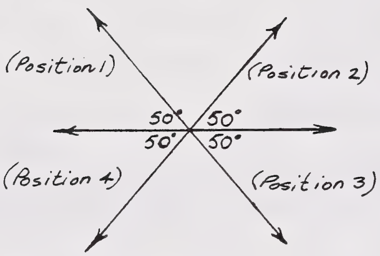
(ii) Rays ______, ______

(iii) Vertex ____________

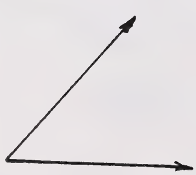
(iv) Measure ____________

(v) Classification ____________

2.

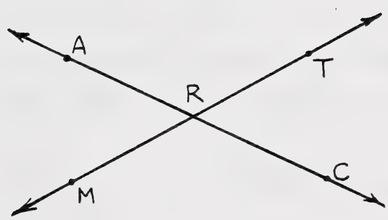


Using your protractor, draw an angle of 50° in the four positions shown in the diagram.

Position 1	Position 2	Position 3	Position 4
			

3. Draw two rays that each make an angle of 75° with $\overrightarrow{AB}$. (Vertex of each angle is at A.)

4.



(a) Name two lines. _____, _____

(b) Name a line segment that is a subset of $\overrightarrow{RM}$. __________

(c) Name the endpoint of $\overrightarrow{RA}$. ________

(d) Name the angle composed of $\overrightarrow{RM}$ and $\overrightarrow{RC}$. ________

(e) Classify $\angle ARM$ as being acute, obtuse, or right. ___________

(f) Classify $\angle ART$ as being acute, obtuse, or right. ___________

(g) Name two rays that are subsets of $\angle TRC$. _____, _____

(h) Name the ray that is a subset of both $\angle ART$ and $\angle TRC$. ________

(i) Name the point that is a subset of both $\angle ART$ and $\angle MRC$. ________

(j) What ray unites with $\overrightarrow{RM}$ to form a line? __________

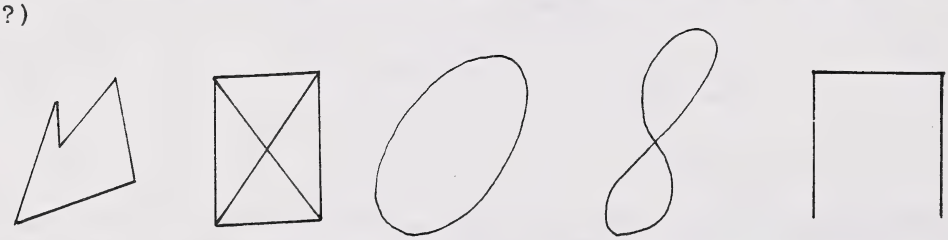
Topic Four: Special Kinds of Curves

> A curve is a set of points which you can draw without lifting your pencil and without retracing any portion of the diagram.

Put a check mark in the blank under each of the following figures that is a curve. (Trace over the figure. Did you have to lift your pencil or retrace?)

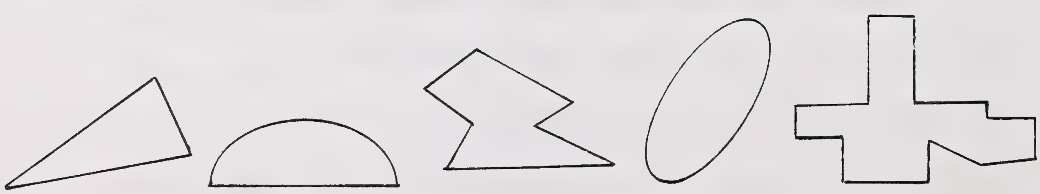
> A simple closed curve is a special kind of curve which begins and ends at the same point and does not cross itself.

Put a check mark in the blank under each figure that is a simple closed curve. (Does the figure start and end at the same point without crossing itself?)

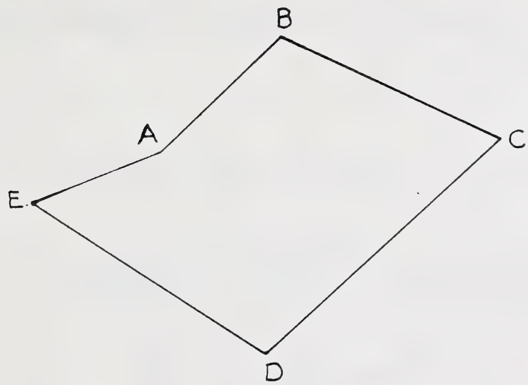


> A polygon is a simple closed curve whose sides are line segments.

Put a check mark in the blank if the figure is a polygon. (Does the figure have sides that are line segments?)



Each line segment which is part of a polygon is called a side of the polygon. Each point where two line segments meet is called a vertex of the polygon. A polygon has the same number of vertices as it has sides. We usually name a polygon by listing the letters at the vertices in a clockwise fashion, starting with the letter closest to the beginning of the alphabet.

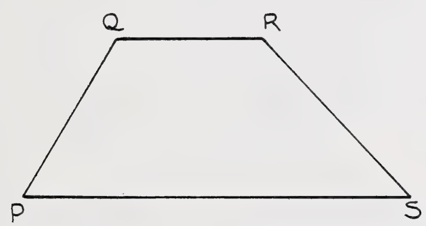


This is polygon ABCDE.

Its five sides are $\overline{AB}$, _____, _____, _____, _____.

Its five vertices are A, _____, _____, _____, _____.

When we join two vertices of a polygon that are not adjacent (ie. they do not lie right next to each other), we obtain a diagonal of the polygon.



Name this polygon. ________________

Draw in the two diagonals of this polygon. The names of these diagonals are $\overline{PR}$ and ________.

Polygons can be classified according to their number of sides. Below is a chart showing the special names assigned to polygons with a certain number of sides. You should learn these names.

Number of sides	Name of polygon
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon

Note that the prefixes of the names for polygons suggest how many sides the polygons have. For example, "tri" suggests "three", "quad" suggests "four", and so on. Think of two other words where "tri" is used as a prefix to suggest the number three.

________________, ________________

Think of two other words where "quad" is used as a prefix to suggest the number four.

________________, ________________

In the space provided below, draw an example of each kind of polygon that is listed.

Pentagon	Quadrilateral	Octagon

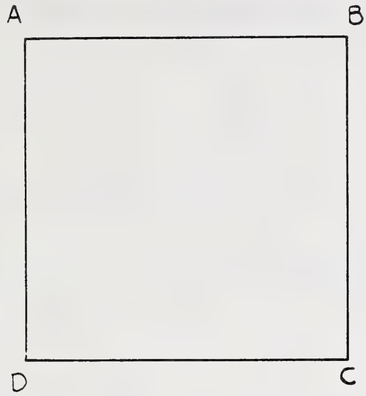
> A regular polygon is a polygon whose sides are all equal in length and angles are equal in measure.

A regular hexagon has how many equal sides and angles? ________________

A regular decagon has how many equal sides and angles? ________________

A regular ________________ has five equal sides and angles.

A regular ________________ has seven equal sides and angles.

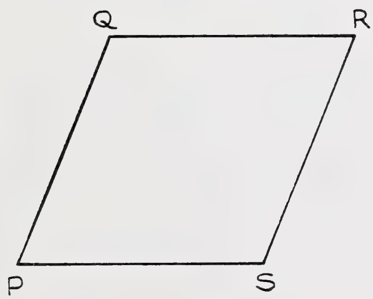


Since this figure has four sides, it is a quadrilateral.

Each side measures __________ cm.

Each angle measure __________°.

ABCD <u>is</u> a regular polygon.



Since this figure has four sides, it is also a ____________________________.

Each side measures __________ cm.

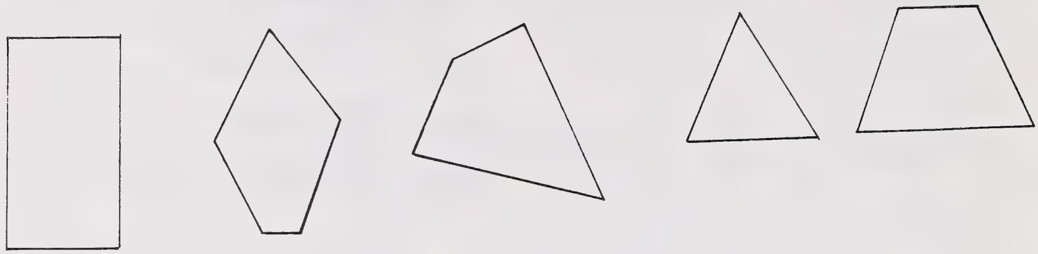
Two angles measure ________° and the other two angles measure ________°.

Since all four angles are <u>not</u> equal in measure, PQRS is <u>not</u> a regular polygon.

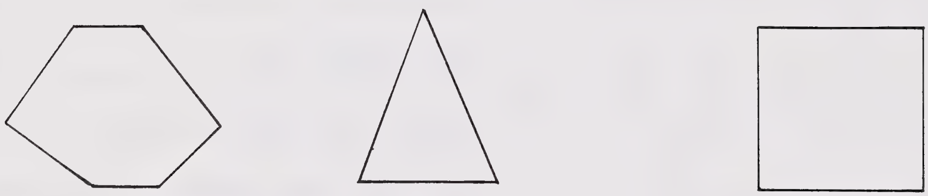
<u>EXERCISE - Special Kinds of Curves</u>

1. Draw a figure that is a simple closed curve but is not a polygon. (Your figure must begin and end at the same point and not cross itself. Its sides can't all be line segments.)

2. Put a check mark under each figure that is a quadrilateral. (Count the number of sides.)

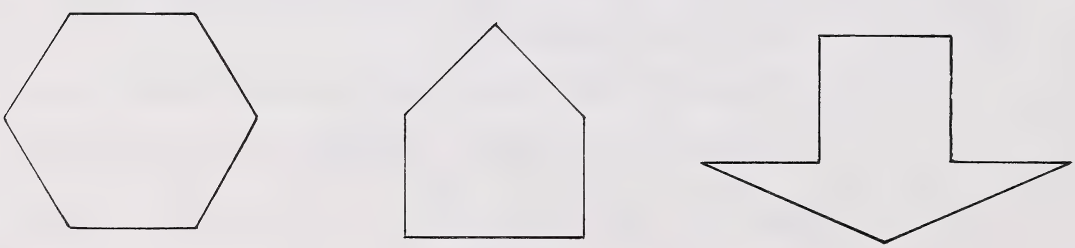


3. State whether each polygon is regular or irregular and then classify it according to its number of sides. (Review chart at bottom of page 21.)

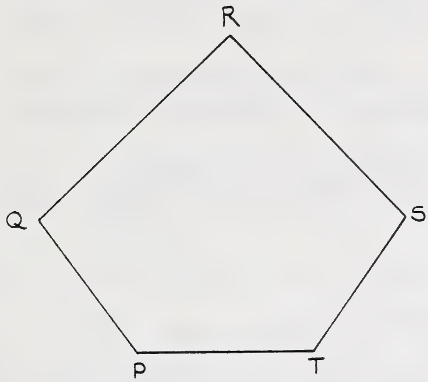


irregular

hexagon



4. Draw in all possible diagonals for the polygon below. Then, name all the sides and all the diagonals.



Sides: $\overline{RS}$, ______

Diagonals: $\overline{RT}$, ______

Topic Five: Triangles

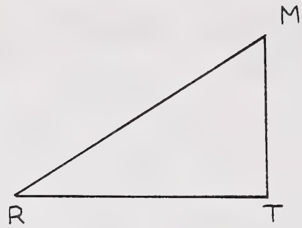
A triangle is a three-sided polygon. A triangle can be drawn by joining three non-collinear points (ie. points that do not all lie in a straight line). Each of these points is called a vertex of the triangle.

A. Parts of a Triangle

Every triangle is made up of three sides that are line segments. These sides have a definite length that can be measured with a ruler. The three sides of a triangle determine three angles. We can only talk about the angles of a triangle if we think of the sides of the triangle as being rays rather than segments. The three angles of a triangle can be measured with a protractor. The three angles together with the three sides of the triangle are called the six parts of the triangle.

A triangle is named by using the symbol Δ which means "triangle" and listing the vertices of the triangle in any order.

EXAMPLE:



(a) This is $\triangle MRT$. Give 5 other ways in which this triangle could be named. $\triangle MTR$, ______________________

(b) The three vertices of this triangle are M , _____, and _____.

(c) $\triangle MRT$ is made up of three line segments. Name each segment and give its measure (give answer to one decimal place).

Segment	Measure
$\overline{MT}$	______
______	______
______	______

(d) $\triangle MRT$ determines three angles. Name each angle and give its measure (to the nearest degree).

Angle	Measure
$\angle MTR$	88°
______	______
______	______

What is the sum of these three measures? __________

This sum should either equal or be very close to 180°. Your sum may differ slightly from 180° because your protractor does not allow you to make exact measurements.

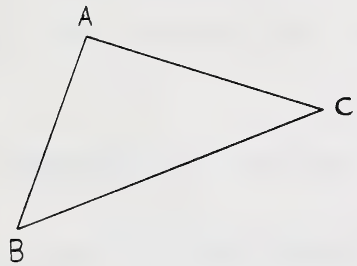
> In general, the sum of the measures of the three angles determined by a triangle is 180°.

B. Classifying Triangles

Triangles can be classified either according to the lengths of their sides or the measures of their angles.

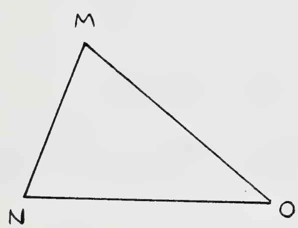
After measuring the sides of a triangle, we can classify it as being scalene, isosceles, or equilateral.

1. A triangle is scalene if all three sides have a different measure.



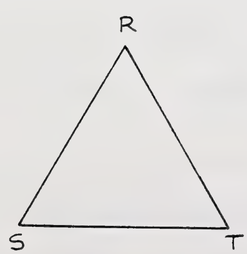
$\triangle ABC$ is a scalene triangle. Its sides measure _____ cm, _____ cm, and _____ cm. (Give answers to one decimal place.)

2. A triangle is isosceles if at least two of its sides have the same measure.



$\triangle MNO$ is an isosceles triangle. Two of its sides measure _______ cm and one side measures _______ cm. (Give answers to one decimal place.)

3. A triangle is equilateral if all three sides have the same measure.

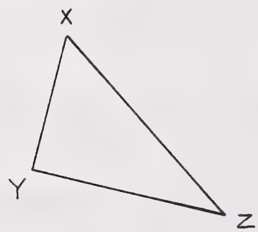


$\triangle RST$ is an equilateral triangle. Each side measures _______ cm. (Give answer to one decimal place.)

Note: Any triangle which is equilateral is also isosceles. (If a triangle has 3 equal sides it certainly has 2 equal sides.)

After measuring the angles of a triangle, we can classify it as being <u>right</u>, <u>acute</u>, or <u>obtuse</u>.

1. A triangle is <u>right</u> if it has one right angle.



ΔXYZ is a right triangle.

Which angle measures $90°$? __________

The other two angles measure _____°

and _____°.

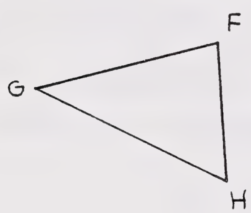
The longest side of a right triangle is called its <u>hypotenuse.</u>

Name the hypotenuse of ΔXYZ. ________

The sides of a right triangle that determine the right angle are called the <u>legs</u> of the right triangle.

The legs of ΔXYZ are $\overline{XY}$ and ________.

2. A triangle is <u>acute</u> if all its angles measure less than $90°$.

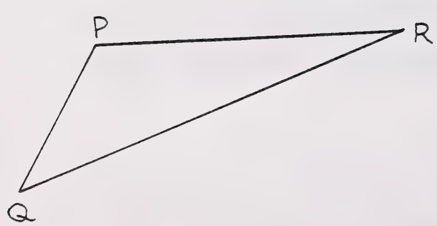


ΔFGH is an acute triangle.

Its three angles measure ___°, ___°,

and ___°.

3. A triangle is <u>obtuse</u> if one of its angles measures more than $90°$.



ΔPQR is an obtuse triangle.

Which angle is obtuse? __________

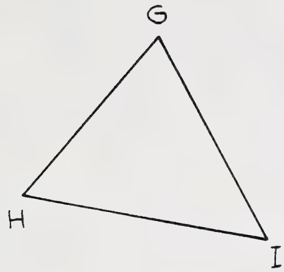
What is the measure of this obtuse angle? __________

The other two angles measure _____° and _____°.

EXERCISE - Triangles

1. In each exercise, name the triangle; name its three sides; state the three vertices; and name the three angles determined by the triangle.

(a)



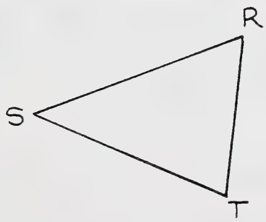
(i) Name of triangle $\triangle GHI$

(ii) Names of sides $\overline{GH}$, ____, ____

(iii) Vertices G, ____, ____

(iv) Names of angles $\angle G$, ____, ____

(b)



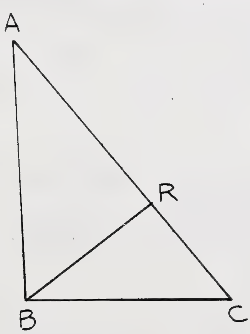
(i) Name of triangle ________

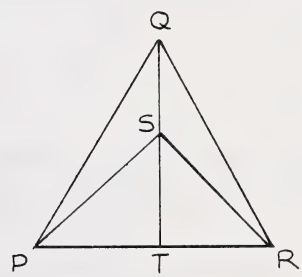
(ii) Names of sides ____, ____, ____

(iii) Vertices ___, ___, ___

(iv) Names of angles ____, ____, ____

2. Name <u>all</u> the triangles in these figures.





(Hint: There are 8 different triangles.)

____________________ $\triangle QPR$, $\triangle QPS$,

____________________ ____________________

____________________ ____________________

3. Draw $\overleftrightarrow{EF}$ and $\overleftrightarrow{CD}$ that intersect at point O. Form $\overline{CE}$, $\overline{ED}$, $\overline{FD}$, and $\overline{CF}$.

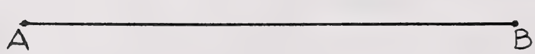
(a) Name four different rays that are shown in the diagram.

$\overrightarrow{OC}$, ______, ______, ______

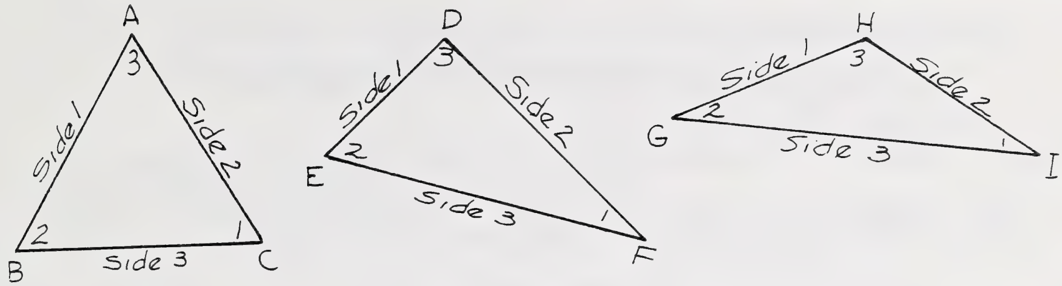
(b) Name eight different triangles that are formed.

$\triangle FCD$, ______, ______, ______, ______, ______, ______, ______

4. Using $\overline{AB}$ below, draw $\triangle ABC$ in which $d(A, B) = 7.6$ cm, $d(A,C) = 5.7$ cm, and $m\angle A = 65°$. (Use your ruler and protractor for this construction.)



5.



(a) For each triangle, measure its sides in centimetres to one decimal place. Record these measurements in the chart below and classify each triangle according to its sides.

Triangle	Measurement of Side			Classification
	1	2	3	
△ABC				
△DEF			5.1 cm	scalene
△GHI				

(b) For each triangle, measure its angles to the nearest degree. Record these measurements in the chart below and classify each triangle according to its angles.

Triangle	Measurement of			Classification
	∠1	∠2	∠3	
△ABC	60°			acute
△DEF				
△GHI				

Topic Six: Other Geometric Terms

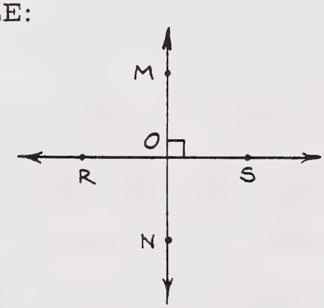
A. Perpendicular Lines

Two lines are PERPENDICULAR if they intersect to form right angles.

The symbol $\perp$ represents the phrase "is perpendicular to". Thus, if line AB is perpendicular to line CD, we can write:

$$\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$$

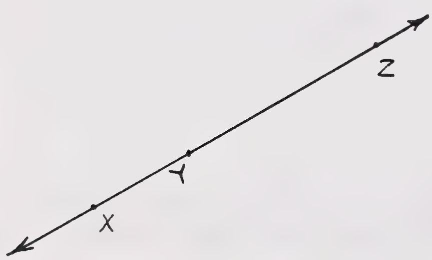
EXAMPLE:



$\overleftrightarrow{MN}$ and $\overleftrightarrow{RS}$ intersect at right angles at point O. (In the diagram, the box drawn at point O indicates that a right angle is formed.) Since $\overleftrightarrow{MN}$ is perpenducular to $\overleftrightarrow{RS}$, we can write:

$$\overleftrightarrow{MN} \perp \overleftrightarrow{RS}$$

$\overleftrightarrow{XZ}$ is drawn below. At point Y, use your protractor to draw a 90° angle. Then draw $\overleftrightarrow{AY}$ such that $\overleftrightarrow{AY}$ is perpendicular to $\overleftrightarrow{XZ}$.



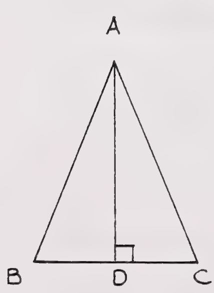
How would you write symbolically that line XZ is perpendicular to line AY?

____

Name two right angles that are formed by these intersecting lines.

__________, __________

Segments are perpendicular if they are subsets of perpendicular lines.



Segment AD is perpendicular to segment BC.

Use symbols to write this. __________

Name two right angles that are formed.

__________, __________

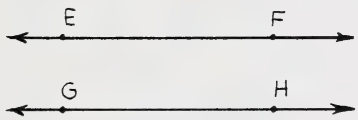
B. Parallel Lines

Two lines are PARALLEL if they lie in the same plane and do not intersect.

The symbol || represents the phrase "is parallel to". Thus, if line AB is parallel to line CD, we can write:

$$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$$

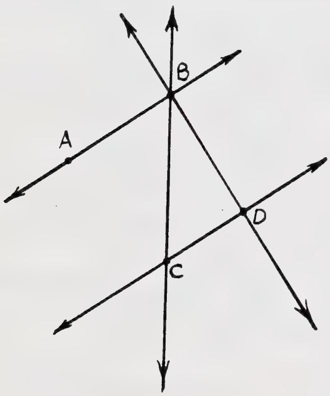
EXAMPLE:



$\overleftrightarrow{EF}$ and $\overleftrightarrow{GH}$ are two lines that lie in the same plane and will never intersect. Since $\overleftrightarrow{EF}$ is parallel to $\overleftrightarrow{GH}$, we can write:

$$\overleftrightarrow{EF} \parallel \overleftrightarrow{GH}$$

Study the diagram below and pick out a pair of parallel lines and a pair of perpendicular lines.



Name the two lines that are parallel.

_____, _____

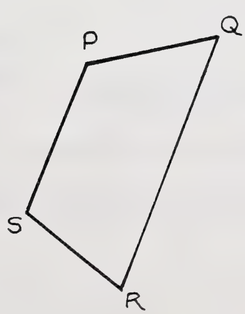
Write this fact symbolically. __________

Name the two lines that are perpendicular.

_____, _____

Write this fact symbolically. __________

Segments are parallel if they are subsets of parallel lines.



Name two segments that are parallel in this figure?

_____, _____

Write this fact symbolically.

____

C. Bisectors

On page 6 of this lesson, we defined the <u>midpoint</u> of a line segment to be the point which lies half-way between the endpoints of the segment.

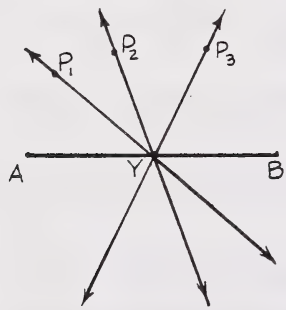
EXAMPLE:

Point Y is the midpoint of $\overline{XZ}$ since $d(X, Y) = d(Y, Z) =$ ______ cm. (to one decimal place)

> Any line which passes through the midpoint of a segment is called a BISECTOR of that segment.

A given segment has an infinite number of bisectors.

EXAMPLE:



What is the midpoint of $\overline{AB}$? ________

$\overleftrightarrow{P_1Y}$, $\overleftrightarrow{P_2Y}$, and $\overleftrightarrow{P_3Y}$ are all bisectors of $\overline{AB}$.

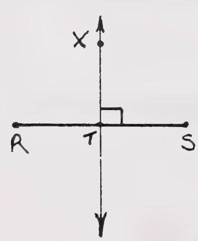
Draw $\overleftrightarrow{P_4Y}$ which is also a bisector of $\overline{AB}$.

For a given segment, we can draw a bisector that cuts the segment at right angles.

> Any line which passes through the midpoint of a segment and cuts the segment at right angles is called the PERPENDICULAR BISECTOR of that segment.

A given segment has only one perpendicular bisector.

EXAMPLE:

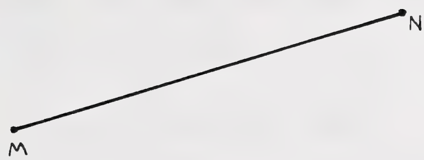


$\overleftrightarrow{XT}$ is the <u>perpendicular bisector</u> of $\overline{RS}$ because

(1) $\overleftrightarrow{XT}$ passes through the midpoint T of $\overline{RS}$.

(2) $\overleftrightarrow{XT}$ is perpendicular to $\overline{RS}$ (ie. $\overleftrightarrow{XT} \perp \overline{RS}$)

Using your ruler and protractor, draw the perpendicular bisector $\overleftrightarrow{ST}$ of $\overline{MN}$ below.

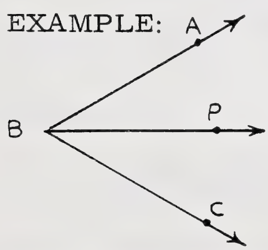


Angles also have bisectors.

The BISECTOR OF AN ANGLE is a ray situated between the arms of the angle having the vertex at its endpoint. This ray forms two angles of equal measure with the arms of the given angle.

A given angle has one and only one bisector.

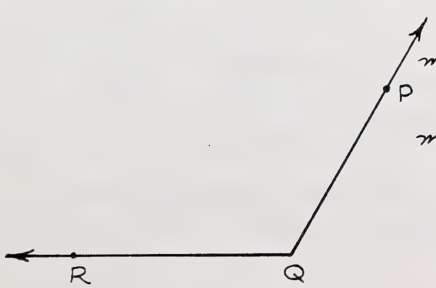
EXAMPLE:



$\overrightarrow{BP}$ is the bisector of $\angle ABC$ because it is situated between the arms of the angle and forms angles of equal measure with the arms.

i.e. $m\angle ABP = m\angle PBC =$ _____ °

Using your protractor, draw bisector $\overrightarrow{QY}$ of $\angle PQR$ below.



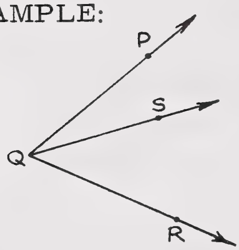
$m\angle PQR =$ _____ °

$m\angle PQY = m\angle$ _____ $=$ _____ °

D. Adjacent Angles

ADJACENT ANGLES are two angles that lie in the same plane and have a common vertex and a common arm which lies between the two other rays.

EXAMPLE:



$\angle PQS$ and $\angle SQR$ are adjacent angles.

Name the common vertex? ________

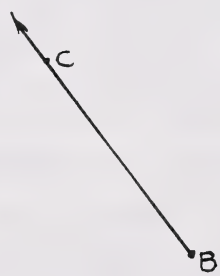
Name the common arm? ________

Using your protractor, draw $\angle ABC$ and $\angle CBD$ below that satisfy the following conditions:

(1) They are adjacent angles with a common vertex B and a common arm $\overrightarrow{BC}$.

(2) $m\angle ABC = 35°$ and $m\angle CBD = 50°$

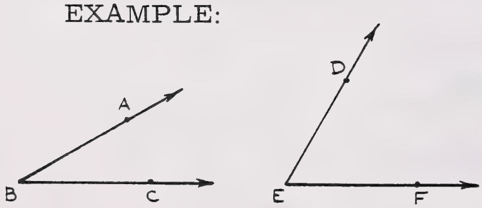
($\overrightarrow{BC}$ has been drawn for you.)



E. Complementary and Supplementary Angles

COMPLEMENTARY ANGLES are two angles such that the sum of their measures is 90°.

EXAMPLE:



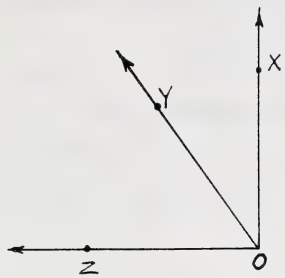
$m\angle ABC$ = _____°, $m\angle DEF$ = _____°

$m\angle ABC + m\angle DEF$ = _____° + _____° = _____°

$\angle ABC$ and $\angle DEF$ are complementary angles since the sum of their measures is 90°.

If two angles are both adjacent and complementary, they form a right angle.

EXAMPLE:



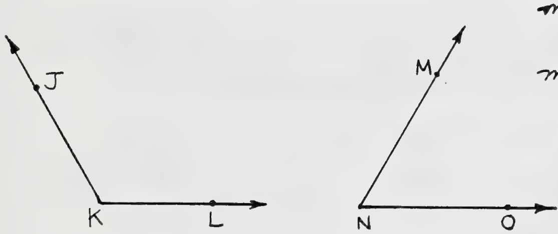
$\angle XOY$ and $\angle YOZ$ are adjacent angles because they have the common vertex _____ and the common arm _____. They are complementary because the sum of their measures is _____°.

$m\angle XOY + m\angle YOZ$ = ___° + ___° = ___°

What kind of angle is $\angle XOZ$? ________________

SUPPLEMENTARY ANGLES are two angles such that the sum of their measures is 180°.

EXAMPLE:



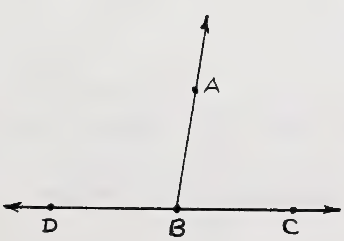
$m\angle JKL$ = _____°; $m\angle MNO$ = _____°

$m\angle JKL + m\angle MNO$ = ___° + ___° = ___°

$\angle JKL$ and $\angle MNO$ are supplementary angles since the sum of their measures is 180°.

If two angles are both adjacent and supplementary, they form a LINEAR PAIR.

EXAMPLE:

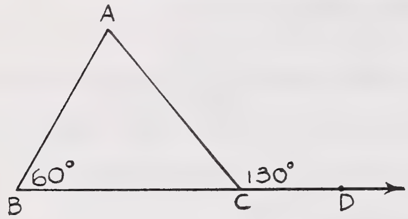


$\angle ABC$ and $\angle$___ are adjacent angles because they have the common vertex ___ and the common arm ___. They are supplementary because the sum of their measures is ___°.

$m\angle ABC + m\angle ABD$ = ___° + ___° = ___°

Note that points D, B, and C lie in the same straight line, so $\angle ABC$ and $\angle ABD$ form a linear pair.

Find the measures of $\angle ACB$ and $\angle BAC$ in $\triangle ABC$ below by using your knowledge of supplementary angles and the sum of the measures of the angles of a triangle.



Since $\angle ACD$ and $\angle ACB$ form a linear pair,

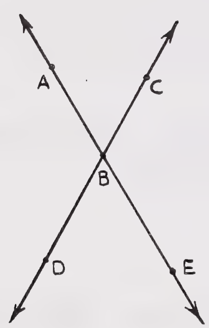
$m\angle ACB =$ ____°.

Since the sum of the measures of the three angles of a triangle is 180°, $m\angle BAC =$ ____°.

F. <u>Vertical Angles</u>

Two angles are VERTICAL ANGLES if they have the same vertex and the sides of one are rays opposite to the sides of the other.

EXAMPLE:



$\angle ABC$ and $\angle DBE$ are vertical angles since they have the common vertex B and $\overrightarrow{BA}$ is opposite $\overrightarrow{BE}$ and $\overrightarrow{BD}$ is opposite $\overrightarrow{BC}$.

$\angle ABD$ and $\angle$___ are also vertical angles. They have the same vertex ___. $\overrightarrow{BA}$ and ___ are opposite rays, while $\overrightarrow{BD}$ and ___ are opposite rays.

Vertical angles always have the same measure. In the diagram above, note that $\angle ABC$ and $\angle CBE$ form a linear pair.

symbol for "therefore" $\longrightarrow \therefore m\angle ABC + m\angle CBE = 180°$

Also, $\angle CBE$ and $\angle EBD$ form a linear pair.

$$\therefore m\angle CBE + m\angle EBD = 180°$$

Thus,

$$m\angle ABC + m\angle CBE = m\angle CBE + m\angle EBD \text{ (since both sums equal } 180°\text{)}$$

We can subtract $m\angle CBE$ from both sides of the equation to obtain:

$$\underline{\underline{m\angle ABC = m\angle EBD}}$$

We have now proved that the two vertical angles $\angle ABC$ and $\angle EBD$ have the same measure.

In a similar fashion, we can prove that the other pair of vertical angles $\angle CBE$ and $\angle ABD$ have the same measure.

$$m\angle CBE + m\angle EBD = \_\_\_^\circ \text{ (since they are a linear pair)}$$

$$m\angle EBD + m\angle ABD = \_\_\_^\circ \text{ (since they are a linear pair)}$$

Thus,

$$m\angle CBE + m\angle EBD = m\angle EBD + m\angle\_\_\_ \text{ (since both sums equal } 180^\circ\text{)}$$

We can subtract $m\angle\_\_\_$ from both sides of the equation to obtain:

$$\underline{\underline{m\angle CBE = m\angle \qquad}}$$

G. <u>Congruency</u>

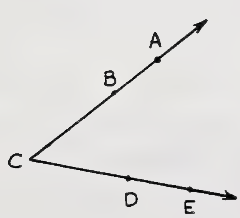
A mathematical statement that contains the relations symbol, =, is called an <u>equation</u>. It tells us that the expressions on either side of the equals sign name the same number. For example, the equation $2 + 7 = 5 + 4$ tells that $2 + 7$ and $5 + 4$ are names for the same number.

If we are to be consistent in our geometry, it follows that the statement $\overline{PQ} = \overline{RS}$ means that $\overline{PQ}$ and $\overline{RS}$ are different names for the same line segment. This can be true only if we have the situation illustrated below.

P and R name the same point.
Q and S name the same point.

Similarly, two angles or two triangles are equal if they consist of the same set of points.

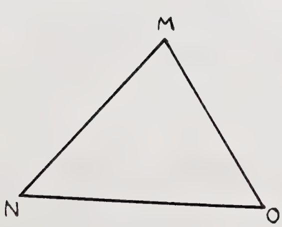
EXAMPLES:



Fill in the blanks with different names for this angle.

$\angle ACE = \angle ECB = \angle\_\_\_\_ = \angle\_\_\_\_ = \angle\_\_\_\_$

(The equals signs imply that these are all names for the <u>same</u> angle.)



Fill in the blanks with different names for this triangle.

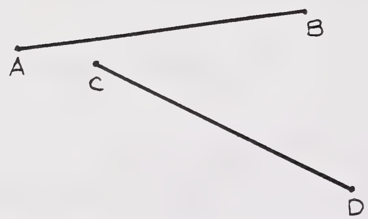
$\Delta MNO = \Delta MON = \Delta\_\_\_\_ = \Delta\_\_\_\_ = \Delta\_\_\_\_$

(The equals signs imply that these are all names for the <u>same</u> triangle.)

Some geometric figures are not identical, but they have the same measure. When this is the case, we say that the figures are CONGRUENT. The symbol $\cong$ represents the phrase "is congruent to".

> Two different line segments or two different angles are <u>congruent</u> if they have the same measure.

EXAMPLE 1:



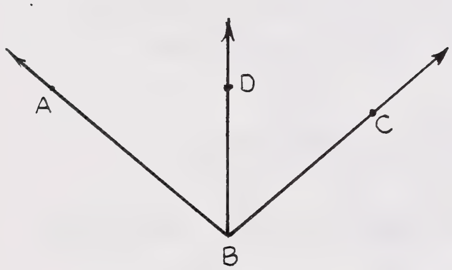
$d(A, B) =$ ___ cm, $d(C, D) =$ ___ cm

Since $\overline{AB}$ and $\overline{CD}$ have the same measure, they are congruent segments. We can write:

$$\overline{AB} \cong \overline{CD}$$

(We <u>cannot</u> say that $\overline{AB} = \overline{CD}$ since $\overline{AB}$ and $\overline{CD}$ are names for two <u>different</u> line segments.)

EXAMPLE 2:



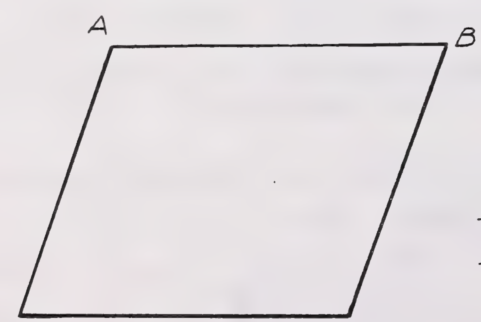
$m\angle ABD =$ ___°, $m\angle DBC =$ ___°

Since $\angle ABD$ and $\angle DBC$ have the same measure, they are congruent angles. We can write:

$$\angle ABD \cong \angle DBC$$

(We <u>cannot</u> say that $\angle ABD = \angle DBC$ since $\angle ABD$ and $\angle DBC$ are names for <u>different</u> angles.)

Use your ruler and protractor to measure the segments and angles in the figure below. Then name two pairs of congruent segments and two pairs of congruent angles. (Give segment lengths to one decimal place.)



$d(A, B) =$ ___ cm, $d(B, D) =$ ___ cm,

$d(C, D) =$ ___ cm, $d(A, C) =$ ___ cm.

Thus, $\overline{AB} \cong$ ___ and ___ $\cong$ ___.

$m\angle A =$ ___°, $m\angle B =$ ___°, $m\angle C =$ ___°,

$m\angle D =$ ___°

Thus,

$\angle A \cong \angle$___ and $\angle$___ $\cong \angle$___

Two simple closed curves are <u>congruent</u> if they fit exactly when placed together. In other words, they have the same size and shape.

We can test for the congruency of two figures by making a tracing on thin paper of one figure and then placing this tracing over the second figure. If the tracing coincides exactly with the second figure, then the two figures must be congruent.

EXAMPLE:

fig. 1

fig. 2

If a tracing is made of figure 1 and this tracing is placed over figure 2, the tracing will correspond exactly with figure 2. Thus, the two figures are congruent.

i.e. fig. 1 $\cong$ fig. 2

In some cases when you make a tracing to test for congruency, you must flip the tracing over before you can make the figures fit exactly.

EXAMPLE:

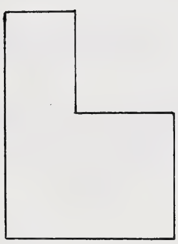


fig. 1

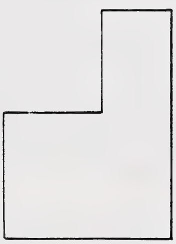


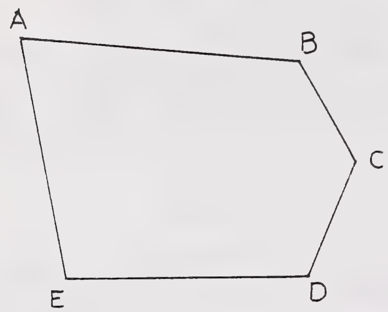
fig. 2

If a tracing is made of figure 1 and this tracing is flipped over and then placed on figure 2, the two figures will correspond exactly. Thus, the two figures are congruent.

i.e. fig. 1 $\cong$ fig. 2

In both examples above, we have a pretty good idea that the figures are congruent without making a tracing. In the first example, if we mentally rotate figure 1 a bit clockwise and slide it over to the right, it seems to correspond exactly with figure 2. In the second example, if we mentally flip figure 1 over on its right-hand edge and move it to the right, it seems to correspond exactly with figure 2.

Study the diagrams below and note that the two polygons are congruent. If polygon ABCDE is flipped over on its edge ED and moved downward, the two figures will coincide.



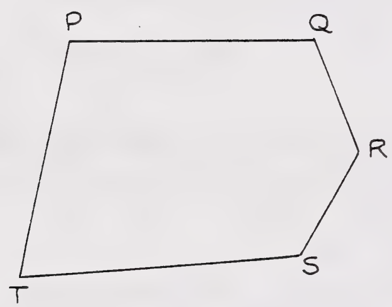
When the two polygons are made to coincide, the vertices will correspond in the following manner:

symbol for "corresponds to"

E $\leftrightarrow$ P, D $\leftrightarrow$ Q, A $\leftrightarrow$ T,

B $\leftrightarrow$ S, C $\leftrightarrow$ R

List the congruencies of the 5 sides and 5 angles.



Sides	Angles
$\overline{ED} \cong \overline{PQ}$	$\angle E \cong \angle P$
$\overline{AE} \cong \overline{TP}$	$\angle D \cong \angle Q$

When we write a sentence stating that these two polygons are congruent, we use names for the polygons that show how the vertices correspond. If we do this, then from the sentence

polygon ABCDE $\cong$ polygon TSRQP,

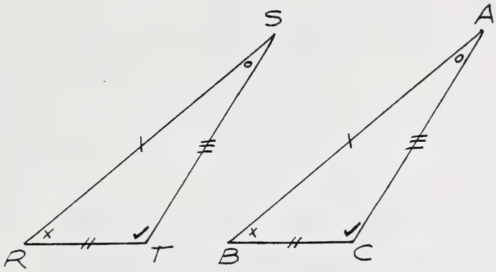
we understand the correspondence A $\leftrightarrow$ T, B $\leftrightarrow$ S, C $\leftrightarrow$ R, D $\leftrightarrow$ Q, and E $\leftrightarrow$ P.

In future work in geometry, you will often be concerned with the congruency of two triangles.

> Two triangles are congruent if corresponding angles are congruent and corresponding sides are congruent.

There are certain markings we can use on congruent triangles to indicate which sides and angles correspond. The symbols | , || , and ||| can be used to mark corresponding sides and the symbols X , ✓, and ∘ can be used to mark corresponding angles.

EXAMPLE:



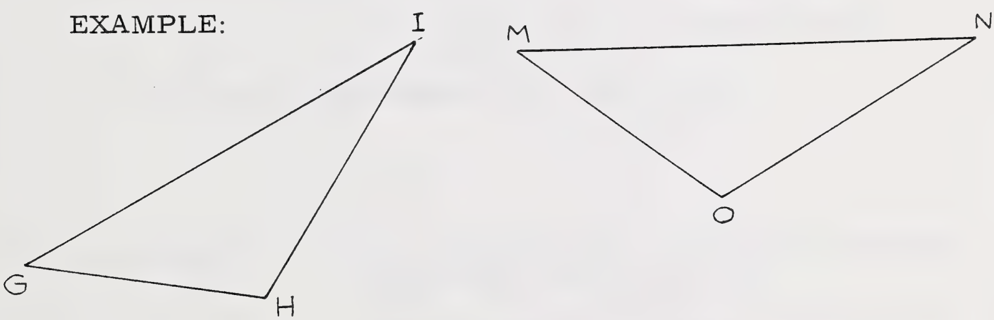
When ΔSRT and ΔABC are made to coincide, $S \longleftrightarrow A$, $R \longleftrightarrow B$, and $T \longleftrightarrow C$. Note how corresponding parts of the triangles are marked with the same symbol. (The symbols | , || , ||| mark corresponding sides and the symbols × , ✓ , ∘ mark corresponding angles.

$$\underbrace{\Delta RST \cong \Delta BAC}$$

This naming reflects the correspondence $R \longleftrightarrow B$, $S \longleftrightarrow A$, $T \longleftrightarrow C$.

When finding the parts of congruent triangles that correspond, you will sometimes have to rotate one of the triangles mentally in order to set up the correspondence.

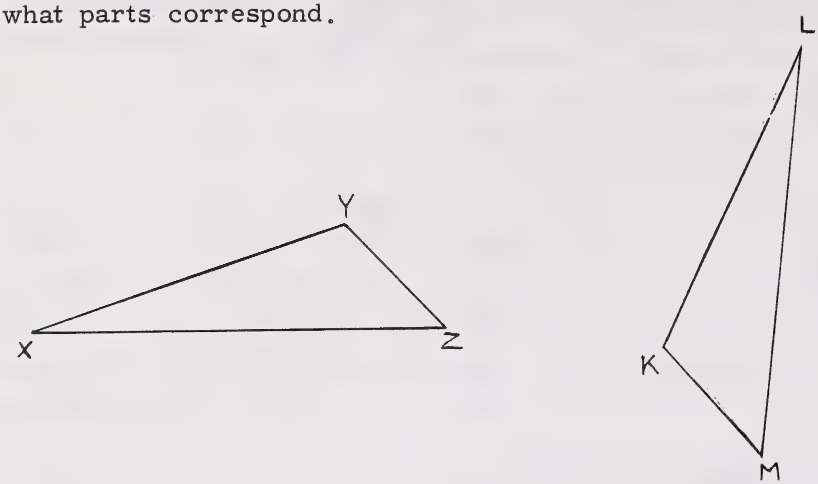
EXAMPLE:



If ΔGHI is rotated slightly clockwise and moved over, it can be made to coincide exactly with ΔMON so that $G \longleftrightarrow M$, $H \longleftrightarrow O$, and $I \longleftrightarrow N$. <u>Mark corresponding sides of the triangle with the symbols | , || , and ||| , and corresponding angles with the symbols ×, ✓, and ∘</u>. Then list the congruencies of the 3 sides and 3 angles below.

Sides	Angles
$\overline{GH} \cong \overline{MO}$	$\angle G \cong \angle M$
____________	____________
____________	____________

Sometimes when you are working with congruent triangles you will have to flip one of the triangles mentally before you can determine what parts correspond.



If ΔXYZ is flipped on its edge YZ and moved over, it can be made to coincide exactly with ΔKLM so that $Y \longleftrightarrow K$, $Z \longleftrightarrow M$, and $X \longleftrightarrow L$. <u>On the diagrams above, mark corresponding sides and angles.</u> Then list the congruencies of the 3 sides and 3 angles below.

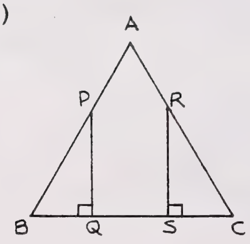
Sides	Angles
$\overline{XY} \cong \overline{LK}$	$\angle X \cong \angle L$
______	______
______	______

<u>Self-correcting Exercise #5</u>

Answers may be found on page 51 of this lesson.

1. A pair of congruent triangles appears in each figure below. In each case, list the congruencies of the 3 sides and 3 angles.

(a)



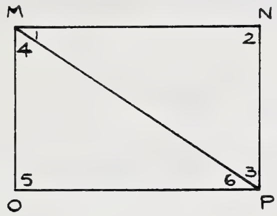
$\Delta PQB \cong \Delta RSC$

(i) $\overline{PQ} \cong$ ______ (iv) $\angle BPQ \cong \angle$ ______

(ii) $\overline{PB} \cong$ ______ (v) $\angle PQB \cong \angle$ ______

(iii) $\overline{BQ} \cong$ ______ (vi) $\angle PBQ \cong \angle$ ______

(b)



$\triangle MNP \cong \triangle POM$

(i) $\overline{MN} \cong$ ______ (iv) $\angle 2 \cong$ ______

(ii) ____ $\cong \overline{MO}$ (v) $\angle 1 \cong$ ______

(iii) $\overline{MP} \cong$ ______ (vi) $\angle 3 \cong$ ______

2. The measure of an angle is 4 times the measure of its complement. Find the measures of the two angles.

Let the measure of the smaller angle be x°.

Then ___° is the measure of the larger angle.

Since the angles are complements, their sum must be ___°.

$$\_\_\_ + \_\_\_ = 90$$

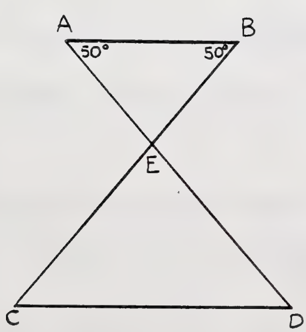
$$\_\_\_ = 90$$

$$x = \_\_\_$$

$$\therefore \_\_x = \_\_\_ \leftarrow \text{larger angle}$$

The two angles measure ___° and ___°.

3. Determine the measure of ∠CED. (Do not use your protractor.)



EXERCISE - Other Geometric Terms

1. Fill in the blanks.

(a) Perpendicular rays form a _______________ angle.

(b) If one angle of a linear pair is obtuse, the other angle is _______________.

(c) If the sum of the measures of two angles is 90°, the angles are called _______________ angles.

(d) $\overleftrightarrow{LM} \perp \overleftrightarrow{PQ}$ means that line LM is _______________ to line PQ.

(e) We use a _______________ to measure angles.

(f) The symbol $m\angle XYZ$ represents the _______________ of _______________ XYZ.

(g) Two triangles are congruent if corresponding _______________ are congruent and corresponding _______________ are congruent.

(h) The three non-collinear points that can be joined to form a triangle are called the _______________ of the triangle.

(i) Any line that passes through the midpoint of a segment is called a _______________ of that segment. If that line also cuts the segment at right angles, it is called the _______________ _______________ of the segment.

(j) Two angles that have a common vertex and a common arm which lies between the other two rays are called _______________ angles.

(k) A(n) _______________ triangle has three congruent sides.

(l) If two angles are both congruent and supplementary, each is a _______________ angle.

(m) The side opposite the 90° angle in a right triangle is called the _______________.

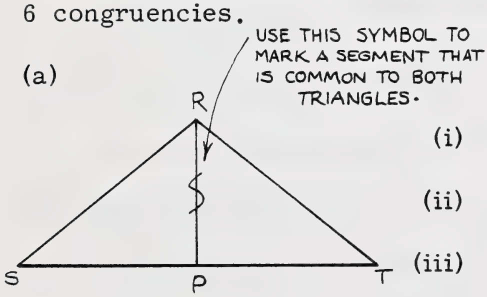
(n) Congruent segments have the same _______________.

(o) If two angles are both adjacent and supplementary, they form a _______________ pair.

2. The measure of an angle is 10° more than the measure of its supplement. Find the measures of the two angles. (See #2 on p. 45).

3. In each figure, find a pair of congruent triangles. Mark the congruent segments and angles on the diagrams and then list these 6 congruencies.

(a)



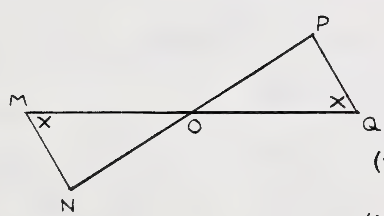
$\triangle$ ______ $\cong \triangle$ ______

(i) $\overline{RS} \cong$ ______ (iv) $\angle SRP \cong$ ______

(ii) $\overline{SP} \cong$ ______ (v) $\angle RPS \cong$ ______

(iii) $\overline{RP} \cong$ ______ (vi) $\angle RSP \cong$ ______

(b)



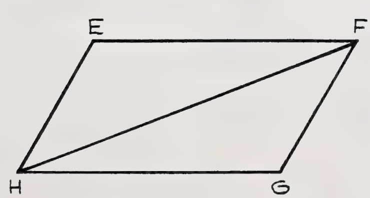
$\triangle$ ______ $\cong \triangle$ ______

(i) ____________ (iv) ____________

(ii) ____________ (v) ____________

(iii) ____________ (vi) ____________

(c)



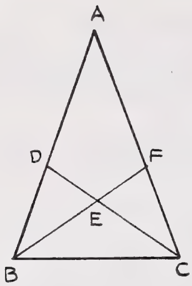
$\triangle$ ______ $\cong \triangle$ ______

(i) ____________ (iv) ____________

(ii) ____________ (v) ____________

(iii) ____________ (vi) ____________

(d)



$\triangle$______ $\cong \triangle$______

(i) __________ (iv) __________

(ii) __________ (v) __________

(iii) __________ (vi) __________

4. Give the meanings of the following symbols.

(a) $\overrightarrow{XY}$ ray xy __________ (b) $\overline{MN}$ __________

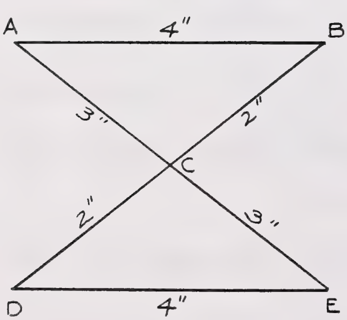
(c) $\angle ABC$ __________ (d) $\triangle PQR$ __________

(e) $\overleftrightarrow{BC}$ __________ (f) $m\angle C$ __________

(g) $\perp$ __________ (h) $\parallel$ __________

(i) d(M, N) __________ (j) $\cong$ __________

5. Study the accompanying figure and decide which statements are true and which are false.



True or False?

(a) $\overline{AB} = \overline{DE}$ __________

(b) $d(A, B) = d(D, E)$ __________

(c) $\overline{AB} \cong \overline{DE}$ __________

(d) $\overline{AC} \cong \overline{BC}$ __________

(e) $\overline{DC} \cong \overline{BC}$ __________

(f) $\angle ACB = \angle DCE$ __________

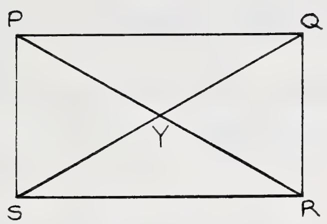
(g) $m\angle ACB = m\angle DCE$ __________

(h) $\triangle ACB = \triangle ECD$ __________

(i) $\triangle ACB \cong \triangle ECD$ __________

(j) $\triangle CDE = \triangle CED$ __________

6. Using the accompanying figure, name each of the following.



(a) a pair of adjacent angles ____, ____

(b) a linear pair of angles ____, ____

(c) a pair of vertical angles ____, ____

(d) a pair of congruent segments ____, ____

(e) a pair of perpendicular segments ____, ____

(f) a pair of parallel segments ____, ____

(g) a right angle ____

(h) a diagonal ____

7. Write the following in symbols.

(a) Triangle ABC is congruent to triangle JKL. ____________

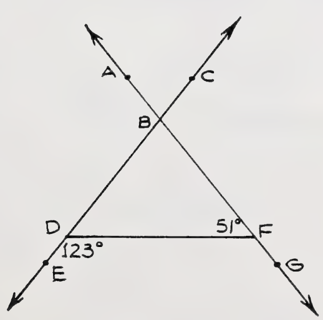
(b) Segment XY is parallel to segment MN. ____________

(c) The measure of angle PQR is 30°. ____________

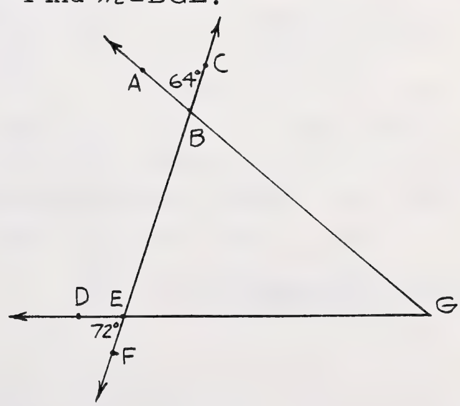
(d) The measure of segment GH is 3 cm. ____________

(e) Ray MN is perpendicular to ray MW. ____________

8. (a) Find $m\angle ABC$.

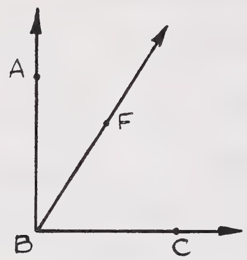


(b) Find $m\angle BGE$.



(Review p. 38 and #3 on page 45.)

9.



(a) Why are $\angle ABF$ and $\angle FBC$ adjacent angles?

____

____

(b) Why are they complementary angles?

____

(c) If $m\angle FBC$ is $y°$, what is $m\angle ABF$? __________

10. Sketch each of the following.

(a) a linear pair of angles

(b) an obtuse-angled triangle

Key to Self-correcting Exercises in Lesson 10

Exercise #1, page 12

1.	(a) $\angle Z$, $\angle XZY$ (or $\angle YZX$)	(b) $\angle Q$, $\angle PQR$ (or $\angle RQP$)	(c) $\angle N$, $\angle MNR$ (or $\angle RNM$)
2.	(a) $\overrightarrow{ZX}$, $\overrightarrow{ZY}$	(b) $\overrightarrow{QP}$, $\overrightarrow{QR}$	(c) $\overrightarrow{NR}$, $\overrightarrow{NM}$

3. $\angle ABD$ (or $\angle DBA$); $\angle DBC$ (or $\angle CBD$); $\angle ABC$ (or $\angle CBA$)

Exercise #2, page 13

1. $m\angle ABC = 65°$ 2. $m\angle RST = 32°$ 3. $m\angle XYZ = 135°$

Exercise #3, page 14

1. $m\angle ABC = 47°$ 2. $m\angle RST = 95°$ 3. $m\angle XYZ = 140°$

Exercise #4, page 16

1. $m\angle TUV = 75°$ acute
2. $m\angle DEF = 102°$ obtuse
3. $m\angle JKL = 90°$ right
4. $m\angle MNO = 85°$ acute

Exercise #5, page 44

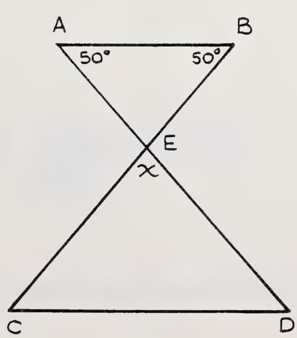
1. (a) (i) $\overline{PQ} \cong \overline{RS}$
(ii) $\overline{PB} \cong \overline{RC}$
(iii) $\overline{BQ} \cong \overline{CS}$
(iv) $\angle BPQ \cong \angle CRS$
(v) $\angle PQB \cong \angle RSC$
(vi) $\angle PBQ \cong \angle RCS$

(b) (i) $\overline{MN} \cong \overline{PO}$
(ii) $\overline{PN} \cong \overline{MO}$
(iii) $\overline{MP} \cong \overline{PM}$
(iv) $\angle 2 \cong \angle 5$
(v) $\angle 1 \cong \angle 6$
(vi) $\angle 3 \cong \angle 4$

2. Let the measure of the smaller angle be $x°$.
Then $(4x)°$ is the measure of the larger angle.
Since the angles are complements, their sum must be $90°$.

$$\begin{aligned} x + 4x &= 90 \\ 5x &= 90 \\ x &= 18 \\ \therefore 4x &= 72 \longleftarrow \text{larger angle} \end{aligned}$$

The two angles measure $18°$ and $72°$.

3. 

Since the sum of the measures of the angles of a triangle is $180°$,

$$\begin{aligned} 50° + 50° + m\angle AEB &= 180° \\ 100° + m\angle AEB &= 180° \\ m\angle AEB &= 80° \end{aligned}$$

Since $\angle AEB$ and $\angle CED$ are vertical angles, they must have the same measure.

Thus, $m\angle CED = 80°$

End of Lesson 10

Basic Algebra and Geometry

Answer Key

For Lessons 1-10

Distance Learning

Alberta
EDUCATION

ANSWER KEY

LESSON 1

Page 1

Is "Alberta" an element of this set? __Yes__ Is "Idaho"? __No__

Is "5"? __No__ Is "Ontario"? __Yes__ Is "Calgary"? __No__

Page 11

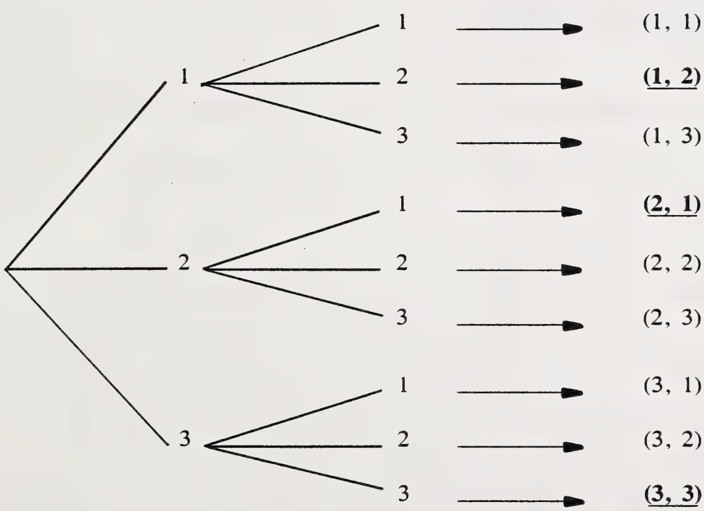
(France, Paris)
France
Paris
(3, 5)
(5, 3)

Page 12

(maple walnut, **butterscotch**)
(**maple walnut**, chocolate)
(strawberry, **chocolate**)

Strawberry ice cream and chocolate topping.

Page 13



Therefore, A × A = {(1, 1), **(1, 2)**, (1, 3), **(2, 1)**, (2, 2), (2, 3), (3, 1), (3, 2), **(3, 3)**.

How many ordered pairs does set A × A contain? __9__

Page 14

Exercise – Sets

1. (a) intersection
   (b) null
   (c) whole
   (d) commas
   (e) subset
   (f) element
   (g) cardinal
   (h) cross, Cartesian, ordered
   (i) finite
   (j) tabulate
   (k) proper
   (l) well-defined
   (m) union
   (n) component

Page 15

2. (a) $A \times B = \{(0, 1), (0, 4), (0, 9), (0, 16), (10, 1), (10, 4), (10, 9), (10, 16), (20, 1), (20, 4), (20, 9), (20, 16)\}$

   (b) $B \times A = \{(1, 0), (1, 10), (1, 20), (4, 0), (4, 10), (4, 20), (9, 0), (9, 10), (9, 20), (16, 0), (16, 10), (16, 20)\}$

   (c) $A \times C = \{(0,3), (0, 12), (10, 3), (10, 12), (20, 3), (20, 12)\}$

   (d) $C \times C = \{(3, 3), (3, 12), (12, 3), (12, 12)\}$

   (e) $C \times B = \{(3, 1), (3, 4), (3, 9), (3, 16), (12, 1), (12, 4), (12, 9), (12, 16)\}$

3. (b) B = {Saturday, Sunday}
   finite

Page 16

   (c) **c** = {11, 12, 13,...}
   infinite

   (d) D = {0, 1, 2,..., 92}
   finite

   (e) E = { }
   null

   (f) F = {a, e, i, o, u}
   finite

   (g) G = {102, 104, 106,...,198}
   finite

   (h) H = {September, April, June, November}
   finite

Page 17

(i) I = { }
null

(j) J = {7, 14, 21,...}
infinite

(k) K = {p, r, a, i, e}
finite

4. (a) nickel <u>yes</u> sixpence <u>no</u>

half-dollar <u>yes</u> quarter <u>yes</u>

$10 bill <u>no</u> franc <u>no</u>

dime <u>yes</u> pound <u>no</u>

(b) 2 <u>no</u> 9 <u>no</u>

4 <u>yes</u> 10 <u>yes</u>

5 <u>no</u> 16 <u>yes</u>

7 <u>no</u> 25 <u>no</u>

Page 18

(c) New York <u>yes</u> Alberta <u>no</u>

Italy <u>no</u> Florida <u>no</u>

Asia <u>No</u> London <u>Yes</u>

Ottawa <u>yes</u> Paris <u>Yes</u>

5. (a) 3
(b) 7
(c) 0
(d) 1
(e) 100

6. (b) {1, 3, 5, 7, 9}
(c) {1, 2, 3, 4, 5}
(d) {4, 5, 6, 7, 8}
(e) {3, 6, 9}
(f) {4, 5, 6, 7, 8, 9, 10}

Page 19

7. (b) $A \cap B = \{2, 3, 6\}$
$A \cup B = \{2, 3, 4, 5, 6, 8, 10\}$

(c) $A \cap B = \{\ \}$ or ϕ
$A \cup B = \{\text{red, blue}\}$

(d) $A \cap B = \{r, s, t\}$
$A \cup B = \{o, p, q, r, s, t, v\}$

(e) $A \cap B = \{t, o, r\}$
$A \cup B = \{t, o, r\}$

8. (b) yes
(c) no
(d) yes
(e) yes
(f) yes
(g) no
(h) no
(i) no
(j) yes

Page 20

9. $A \times B$ = {(white, brown), (white, navy), (white, black), (black, brown), (black, navy), (black, black), (yellow, brown), (yellow, navy), (yellow, black)}

The first component of each ordered pair belongs to set **A** and represents the colour of **sweater** you are wearing.

The second component of each ordered pair belongs to set **B** and represents the colour of **slacks** you are wearing.

The ordered pair (white, brown) represents an outfit composed of a white **sweater** and brown **slacks**.

10. { }, {7}, {11}, {14}, {7, 11}, {7, 14}, {11, 14}, {7, 11, 14}

Which subset is not a proper subset? **{7, 11, 14}**

Page 21

What capital letter has been used to represent this set? **N**

What do the three dots mean? **"and so on"**

Does 5 belong to set N? **Yes**
Does −2? **No**
Does 2160? **Yes**
Does 0? **No**
Does 1.7? **No**
Does 3½? **No**
Does 100? **Yes**

What capital letter has been used to represent this set? W

Does 5 belong to set W? Yes

Does −2? No

Does 2160? Yes

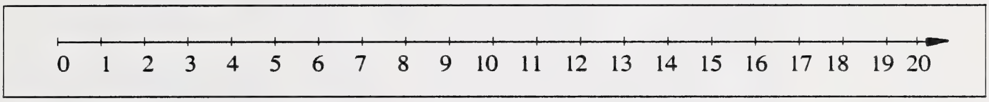
Does 0? Yes

Does 1.7? No

Does 3½? No

Does 100? Yes

Page 22



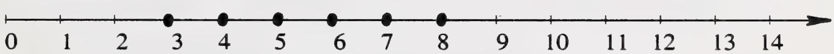
Page 24

2. $\{x \mid x \leq 9, x \in W\}$

"The set of all x such that x is less than or equal to 9 and x is a whole number."

Page 27

M = {3, 4, 5, 6, 7, 8}



Page 29

Exercise – Natural Numbers and Whole Numbers

1. (a) < (b) >
   (c) < (d) >

2.

	a = b	a < b	a > b
(a)			✓
(b)		✓	
(c)	✓		
(d)			✓
(e)	✓		
(f)		✓	
(g)		✓	

3. (b) $<$, left
   (c) $>$, right
   (d) $<$, left
   (e) $>$, right

Page 30

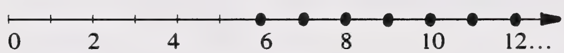
4. (a) true
   (b) true
   (c) false
   (d) false
   (e) true
   (f) true
   (g) true
   (h) false

5. (b) $x < 23$
   (c) $4 < x < 12$
   (d) $x \geq 14$
   (e) $3 \leq x < 21$
   (f) $7 < x < 13$
   (g) $1 < x \leq 15$
   (h) $25 \leq x \leq 50$
   (i) $x > 100$

Page 31

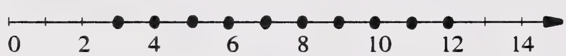
6. (b) x is less than 33
   (c) x is greater than or equal to 2 and x is less than 50
   (d) x is greater than 10 and less than 25
   (e) x is greater than or equal to 14
   (f) x is greater than or equal to 90 and less than or equal to 100

7. (b) $\{x \mid x > 5, x \in W\}$
   $\{6, 7, 8, \ldots\}$

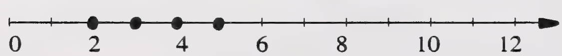


Page 32

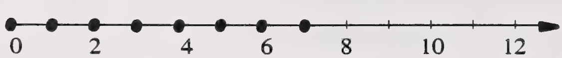
   (c) $\{x \mid 3 \leq x < 13, x \in W\}$
   $\{3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$



   (d) $\{x \mid 1 < x < 6, x \in W\}$
   $\{2, 3, 4, 5\}$



   (e) $\{x \mid x \leq 7, x \in W\}$
   $\{0, 1, 2, 3, 4, 5, 6, 7\}$



END OF LESSON 1

LESSON 2

Page 1

1. In the number sentence $1 + 9 = 10$, the addends are $\underline{1}$ and $\underline{9}$, and the sum is $\underline{10}$. The symbol $\underline{+}$ is used to indicate that the operation is addition.

2. 25

3. (ii) 6 and 2
   (iii) 3 and 5
   (iv) 4 and 4

4. addends, sum

Page 2

1. In the number sentence $3 \times 6 = 18$, the factors are $\underline{3}$ and $\underline{6}$, and the product is $\underline{18}$. The symbol $\underline{\times}$ is used to indicate that the operation is multiplication.

2. 56

3. (ii) 2 and 32
   (iii) 4 and 16
   (iv) 8 and 8

4. factors, product

Page 3

$$\begin{array}{r} 8762 \\ \underline{3568} \\ 12\ 330 \end{array} \qquad \begin{array}{r} 3568 \\ \underline{8762} \\ 12\ 330 \end{array}$$

yes

Page 4

$$\begin{array}{r} 420 \\ \underline{305} \\ 2100 \\ 0000 \\ \underline{1260} \\ 128100 \end{array} \qquad \begin{array}{r} 305 \\ \underline{420} \\ 000 \\ 610 \\ \underline{1220} \\ 128100 \end{array}$$

yes

Page 6

2. $(13 + 5) + 8$	$=$	$13 + (5 + 8)$
$18 + 8$	|	$13 + 13$
26		26
3. $7 + (7 + 4)$	$=$	$(7 + 7) + 4$
$7 + 11$	|	$14 + 4$
18		18
4. $16 + (4 + 10)$	$=$	$(16 + 4) + 10$
$16 + 14$	|	$20 + 10$
30		30

Page 7

1. $50 + 48$
   98

2. $25 + (28 + 32)$
   $25 + 60$
   85

3. $17 + (152 + 48)$
   $17 + 200$
   217

4. $(149 + 11) + 38$
   $160 + 38$
   198

Page 8

2. 48×5	|	6×40
240		240
3. 8×32	|	64×4
256		256
4. 9×77	|	63×11
693		693

1. $13 \times 4 \times 250$
   $= 13 \times (4 \times 250)$
   $= 13 \times 1000$
   $= 13\ 000$

2. $50 \times 2 \times 39$
   $= (50 \times 2) \times 39$
   $= 100 \times 39$
   $= 3900$

3. $25 \times 4 \times 15$
   $(25 \times 4) \times 15$
   $= 100 \times 15$
   $= 1500$

4. $13 \times 8 \times 25$
   $= 13 \times (8 \times 25)$
   $= 13 \times 200$
   $= 2600$

Page 10

1. $(113 + 7) + 25$
$= 120 + 25$
$= 145$

2. $(20 \times 5) \times 23$
$= 100 \times 23$
$= 2300$

3. $(36 + 48) + (48 + 12)$
$= 100 + 60$
$= 160$

4. $(15 \times 2) \times 7$
$= 30 \times 7$
$= 210$

Page 12

1. 60
2. $2(98 + 2) = 2(100) = 200$
3. $6(189 + 11) = 6(200) = 1200$
4. $8(96 + 4) = 8(100) = 800$
5. $4(31 + 9) = 4(40) = 160$

Page 13

$= (4 \times 100) + (4 \times 50) + (4 \times 3)$
$= \mathbf{400 + 200 + 12}$
$= \mathbf{612}$

1. $(7 \times 10) + (7 \times 9) = 70 + 63 = 133$
2. $8(30 + 5) = (8 \times 30) + (8 \times 5) = 240 + 40 = 280$
3. $(3 \times 400) + (3 \times 30) + (3 \times 1)$
$= 1200 + 90 + 3$
$= 1293$

1. $(3 + 8)\,a = 11a$
2. $(6 + 4)\,x = 10x$
3. $(17 + 1)\,c = 18c$
4. $(1 + 6)\,a = 7a$
5. $(7 + 15)\,ab = 22ab$
6. $(19 + 15 + 3)\,x = 37x$

Page 14

Add 10 and 55. **65**	**Yes**
Add 0 and 50. **50**	**Yes**
Add 615 and 1213. **1828**	**Yes**

No

Multiply 6 and 3. **18**	**Yes**
Multiply 15 and 10. **150**	**Yes**
Multiply 12 and 12. **144**	**Yes**

Page 15 No

Exercise – Properties of W under Addition and Multiplication

1. (a) commutative
   (b) binary
   (c) distributive
   (d) 3 + (5 + 7)
   (e) whole
   (f) commutative, addition
   (g) (b × c)
   (h) addition, multiplication
   (i) associative

Page 16

   (j) commutative, associative
   (k) multiplication, addition
   (l) distributive

2. (a)
$$\begin{array}{r} 135 \\ \underline{\times 27} \\ 945 \\ 270 \\ \hline 3645 \end{array} \qquad \begin{array}{r} 27 \\ \underline{\times 135} \\ 135 \\ 81 \\ \underline{27} \\ 3645 \end{array}$$
   (b)
$$\begin{array}{r} 506 \\ \underline{\times 702} \\ 1012 \\ 0000 \\ \underline{3542} \\ 355\ 212 \end{array} \qquad \begin{array}{r} 702 \\ \underline{506} \\ 4212 \\ 0000 \\ \underline{3510} \\ 355\ 212 \end{array}$$

3. $3(7 + 11) = 3(7) + 3(11)$

4. (a) $4 + 8 = 8 + 4$ | (b) $4 \times 8 = 8 \times 4$

5. (b) $(3 + 9 + 17)xy = 29xy$
   (c) $(3 + 5 + 1)a = 9a$
   (d) $(4 + 4)xy = 8xy$
   (e) $(2 + 3 + 5 + 9)b = 19b$
   (f) $(1 + 5 + 2)y = 8y$

Page 17

6. (b) associative property of multiplication
   (d) associative property of addition
   (e) commutative property of multiplication
   (f) closure of property of multiplication
   (g) commutative property of addition
   (h) commutative property of addition
   (i) commutative property of multiplication
   (j) associative property of multiplication
   (k) commutative property of multiplication

(l) distributive property
(m) commutative property of multiplication'
(n) distributive property
(o) commutative property of addition
(p) closure property of multiplication
(q) commutative property of multiplication
(r) associative property of multiplication

Page 18

1. In the number sentence $9 - 3 = 6$, the minuend is **9**, the subtrahend is **3**, and the difference is **6.** The symbol $-$ is used to indicate the operation is subtraction.

2. 3

3. (ii) $6 - 4 = 2$
   (iii) $4 - 2 = 2$
   (iv) $12 - 10 = 2$

4. In the number sentence $12 - 5 = 7$, 12 is called the **minuend,** 5 is called the **subtrahend,** and 7 is called the **difference.**

5. A subtraction is possible in set W only if the **minuend** is larger than the **subtrahend.**

Page 19

(i) $(8 + 7) - 7 = \mathbf{8}$	(ii) $(15 - 9) + 9 = \mathbf{14}$
(iii) $(\mathbf{17} - \mathbf{5}) + \mathbf{5} = 17$	(iv) $(14 + 3) - \mathbf{3} = 14$
(v) $(16 - \mathbf{8}) + 8 = 16$	(vi) $(\mathbf{21} - 7) + 7 = 21$
(viii) $(\mathbf{35} + 6) -6 = 35$	(viii) $(15 + 12) - \mathbf{12} = 15$

Page 20

1. 2
2. 79, 125
3. the number which must be added to 6 to make 23
4. $18 - 6 = \mathbf{12}$ because $6 + \mathbf{12} = 18$
5. $26 - 9 = \mathbf{17}$ because $\mathbf{17} + \mathbf{9} = 26$

2. $6 + 1 = 7$
3. $17 + 19 = 36$
5. $35 + y = 42$
6. $10 + n = 16$

Page 21

(i) no	(ii) no
(iii) no	(iv) no

$9 - (7 - 1)$
$= 9 - \mathbf{6}$
$= \mathbf{3}$

$(9 - 7) - 1$
$= \mathbf{2} - 1$
$= \mathbf{1}$

Are the two sides equal? **No**

Page 22

$5(6 - 3)$
$= 5 \times \mathbf{3}$
$= \mathbf{15}$

$(5 \times 6) - (5 \times 3)$
$= \mathbf{30} - \mathbf{15}$
$= \mathbf{15}$

Are the two sides equal? **Yes**

Page 23

2. $2x - 2y$
3. $3a + 3b + 3c$
4. $6a + 6b - 6c$
5. $5x - 5y - 5z$

2. $7(8 - 3) = 7(5) = 35$
3. $(4 - 2)16 = 2(16) = 32$
4. $40(7 - 5) = 40(2) = 80$
5. $23(8 - 7) = 23(1) = 23$

2. $x(14 - 7) = 7x$
3. $ab(35 - 25) = 10ab$
4. $(7 + 3 - 2)x^2 = 8x^2$
5. $(13 - 7 - 5)y = y$

Page 24

1. 16 − 9
2. 27 − 35
3. 17 − 17 **W**
4. 62 − 70
5. 33 − 29 **W**
6. 6 − 9

The result is not a whole number in the above questions numbered **2, 4,** and **6.**

Page 25

1. In the number sentence $15 \div 3 = 5$, the divisor is **3,** the dividend is **15**, and the quotient is **5.** The symbol $\div$ is used to indicate the operation is division.

2. 4

3. (ii) $21 \div 7 = 3$
   (iii) $24 \div 8 = 3$
   (iv) $18 \div 6 = 3$

   Or any other example that will give a quotient of 3.

4. In the number sentence $24 \div 6 = 4$, 24 is called the **dividend**, 6 is called the **divisor** and 4 is called the **quotient**.

Page 26

(i) 6	(ii) 6
(iii) 4	(iv) 4
(v) 3	(vi) 24
(vii) 5	(viii) 3

Page 27

1. 3
2. 25, 125
3. number which must be multiplied by a to make b.
4. 4, 4
5. 9, 9×7

2. $9 \times 10 = 90$
3. $8 \times 4 = 32$
5. $7 \times n = 14$
6. $4 \times y = 32$

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(i) no
(ii) no
(iii) no

$36 \div (6 \div 3)$	$(36 \div 6) \div 3$
$= 36 \div 2$	$= 6 \div 3$
$= 18$	$= 2$

Are the two sides equal? **No**

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1. **16** $\div$ 9	4. $18 \div 9$ **W**
2. $45 \div 5$ **W**	5. $3 \div 7$
3. $14 \div 4$	6. $32 \div 8$ **W**

The result is not a whole number in the above questions numbered **1, 3,** and **5.**

Page 30 **Exercise – Properties of W under the Four Fundamental Operations**

1. (a) subtraction, division
   (b) division
   (c) 4, 10

(d) associative
(e) addition
(f) 3, 6
(g) subtraction
(h) closed, division
(i) multiplication, subtraction

2. (b) true

(c)	Addition is associative	true
(d)	Subtraction is not associative	false
(e)	Subtraction is not commutative	false
(f)	W is closed under multiplication	true
(g)	Division is not commutative	false
(i)	W is not closed under subtraction	false
(j)	Distributive property	true
(k)	Multiplication is commutative	true
(l)	Addition is commutative	true
(m)	Subtraction is not commutative	false
(n)	Subtraction is not associative	false
(o)	Multiplication is associative	true
(p)	Distributive property	true

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(a) $5 + 7$ yes
(b) $5 - 7$ no
(c) $7 - 5$ yes
(d) $3 \div 6$ no
(e) 9×7 yes
(f) 7×9 yes
(g) $9 \div 3$ yes
(h) $9 \div 4$ no
(i) $4 - 8$ no
(j) $4 + 8$ yes
(k) $6 - 6$ yes
(l) $4 \div 9$ no

4. (b) $E = \{2, 4, 6, \ldots\}$
$2 + 6 = \mathbf{8}$
$14 + 6 = \mathbf{20}$
$12 + 28 = \mathbf{40}$

Are the results always members of E? **Yes**
Why? **They are all even whole numbers.**

Is set E closed under addition? **Yes.**

(c) $D = \{5, 6, 7, \ldots, 20\}$
$5 + 6 = \mathbf{11}$
$7 + 16 = \mathbf{23}$
$18 + 19 = \mathbf{37}$

Are the results always members of D? No
Why? **Some sums exceed 20.**

Is set D closed under addition? **No**

(d) $\mathbf{B} = \{5, 10, 15, \ldots\}$
$5 + 10 = \mathbf{15}$
$10 + 15 = \mathbf{25}$
$20 + 25 = \mathbf{45}$

Are the results always members of B? Yes
Why? **They are all multiples of 5.**

Is set B closed under addition? **Yes**

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5. (a) Closing the door
   (b) Taking off your shoes
   (c) Adding 8
   (d) Turning off the tap
   (e) Dividing by 7
   (f) Multiplying by 4

6. (b)

$$\begin{array}{r} 356 \\ -127 \\ \hline 229 \end{array} \qquad \text{Check} \quad \begin{array}{r} 127 \\ \text{(add)}229 \\ \hline 356 \end{array}$$

(c)

$$\begin{array}{r} 60\,005 \\ -49\,898 \\ \hline 10\,107 \end{array} \qquad \begin{array}{r} 49\,898 \\ \text{(add)}10\,107 \\ \hline 60\,005 \end{array}$$

(d)

$$\begin{array}{r} 40\,306 \\ -39\,428 \\ \hline 878 \end{array} \qquad \begin{array}{r} 39\,428 \\ 878 \\ \hline 40\,306 \end{array}$$

7. (b)

$$\begin{array}{r} 398 \\ 42\overline{)16716} \\ \underline{126} \\ 411 \\ \underline{378} \\ 336 \\ \underline{336} \\ 0 \end{array} \qquad \text{Check} \quad \begin{array}{r} 398 \\ \underline{42} \\ 796 \\ \underline{1592} \\ 16716 \end{array}$$

(c)

$$\begin{array}{r} 21 \\ 412\overline{)8652} \\ \underline{824} \\ 412 \\ \underline{412} \\ 0 \end{array} \qquad \begin{array}{r} 412 \\ \underline{21} \\ 412 \\ \underline{824} \\ 8652 \end{array}$$

(d)

$$\begin{array}{r} 4002 \\ 19\overline{)76038} \\ \underline{76} \\ 0038 \\ \underline{38} \\ 0 \end{array} \qquad \begin{array}{r} 4002 \\ \underline{19} \\ 36018 \\ \underline{4002} \\ 76038 \end{array}$$

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8. (a) 4
   (b) no whole number
   (c) 6
   (d) 7
   (e) 0
   (f) 12
   (g) no whole number
   (h) no whole number
   (i) 6
   (j) 36

$18 + 0 = \mathbf{18}$
$\mathbf{0} + 13 = 13$
$43 + 0 = \mathbf{43}$
$0 + \mathbf{8} = 8$

$0 + 17 = \mathbf{17}$
$15 + \mathbf{0} = 15$
$\mathbf{44} + 0 = 44$
$0 + \mathbf{200} = 200$

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$\mathbf{23 - 0 = 23}$
$16 - \mathbf{0} = 16$
$85 - 0 = \mathbf{85}$
$\mathbf{4} - 0 = 4$

$14 - \mathbf{0} = 14$
$32 - 0 = \mathbf{32}$
$\mathbf{a} - 0 = \mathbf{a}$ (a can be any integer)
$2 - \mathbf{0} = 2$

$7 - 7 = \mathbf{0}$
$8 - \mathbf{8} = 0$
$\mathbf{3} - 3 = 0$
$16 - 16 = \mathbf{0}$

$12 - \mathbf{12} = 0$
$\mathbf{33} - 33 = 0$
$4 - \mathbf{4} = 0$
$112 - 112 = \mathbf{0}$

$\mathbf{0 \times 12 = 0}$
$\mathbf{0} \times 7 = 0$
$4 \times \mathbf{0} = 0$
$3 \times 0 \times 8 = \mathbf{0}$

$\mathbf{0} \times 35 = 0$
$6 \times \mathbf{0} = 0$
$2 \times \mathbf{0} \times 3 = 0$
$5 \times 8 \times \mathbf{0} = 0$

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$0 \div 9 = \mathbf{0}$
$0 \div 2 = \mathbf{0}$
$\mathbf{0} \div 3 = 0$

$\mathbf{0} \div 4 = 0$
$\left.\begin{array}{l} 0 \div \mathbf{a} = 0 \\ 0 \div \mathbf{b} = 0 \end{array}\right\}$ (a and b can be any integer)

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$15 \div \mathbf{0}$ is undefined
$4 \div \mathbf{0}$ is undefined
$15 \div 0$ is **undefined**
$\mathbf{a} \div 0$ is undefined (a can be any integer)

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$1 \times 12 = \mathbf{12}$
$4 \times 1 = \mathbf{4}$
$\mathbf{1} \times 3 = 3$
$\mathbf{100} \times 1 = 100$

$\mathbf{5} \times \mathbf{1} = 5$
$1 \times \mathbf{14} = 14$
$\mathbf{52} \times 1 = 52$
$1 \times \mathbf{0} = 0$

$7 \div 1 = \mathbf{7}$
$12 \div \mathbf{1} = 12$
$\mathbf{3} \div 1 = 3$

$\mathbf{0} \div 1 = 0$
$5 \div \mathbf{1} = 5$
$2 \div 1 = \mathbf{2}$

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$12 \div 12 = \mathbf{1}$
$2 \div 2 = \mathbf{1}$
$7 \div \mathbf{7} = 1$

$\mathbf{3} \div 3 = 1$
$100 \div \mathbf{100} = 1$
$9 \div 9 = \mathbf{1}$

Page 41 **Exercise – Properties of Zero and One**

1. (a) identity, addition, additive
   (b) 0
   (c) 0
   (d) undefined
   (e) 1
   (f) identity, multiplication, multiplicative
   (g) 0
   (h) 1, 0
   (i) 0
   (j) 1
   (k) equal

2.

	True or False	Correction
(c)	false	$7 \times 0 = 0$
(d)	true	
(e)	false	$5 + 0 = 5$
(f)	false	$8 \div 8 = 1$
(g)	false	$0 \div 5 = 0$
(h)	true	
(i)	false	$3 - 0 = 3$
(j)	true	
(k)	false	$14 \div 1 = 14$

3. (a) 16 (b) 4 (c) 0
   (d) 0 (e) 0 (f) undefined
   (g) 45 (h) 1 (i) 31
   (j) 0 (k) 4 (l) $0 + 3 = 3$
   (m) $4 \times 0 = 0$ (n) $6 - 4 = 2$ (o) $14 \times 0 = 0$
   (p) $3\ (7 \div 0) =$ undefined (q) $0 \div 3 = 0$ (r) $1 \times 8 = 8$
   (s) $1 \div 0 =$ undefined (t) $0 \div 10 = 0$ (u) $10 \div 10 = 1$
   (v) $9 \div 9 = 1$ (w) $6 \div 3 = 2$

   (x) $\frac{10}{0} + 1 =$ undefined

   (y) $\frac{4}{0} =$ undefined

   (z) $\frac{4}{1} = 4$

End of Lesson 2

LESSON 3

Page 1

1. 16 − 12 **W**
2. 1 − 2
3. 8 − 0 **W**
4. 0 − 12
5. 14 − 13 **W**
6. 70 − 90

Page 2

Does −6 belong to set I? **Yes** Does 6? **Yes** Does ½? **No** Does 3.2? **No** Does 0? **Yes** Does −100? **Yes** Does 1⅔? **No**

above zero? **+75**, 10° below zero? **−10**

a less of \$100? **−\$100**, a profit of \$9 **\$9**

1. −20
2. +5
3. +30
4. −500
5. +150
6. −1800
7. −5
8. +1500
9. −110
10. +100

Page 3

2. left of the origin
3. right of the origin
4. right of the origin
5. left of the origin
6. right of the origin

Page 4

2. 4 , left of origin
3. 100 , right of origin
4. 2 , left of origin
5. 7 , right of origin
6. a , left of origin

−2 and **+2** are opposites
+3 and **−3** are opposites
12 and **−12** are opposites

Page 5

1. $|-3|$ and equals 3
2. $|+5|$ and equals 5
3. $|-12|$ and equals 12

What two integers both have an absolute value of 8? **+8, −8**

Page 6

Since 2 is to the **right** of -4, $2 > -4$.
Since -3 is to the **right** of -6, $-3 > -6$.

Since -3 is to the **left** of -2, $-3 < -2$
Since -4 is to the **left**of 2, $-4 < 2$.

Page 7

Does $-5\frac{1}{2}$ belong to this set? **No** Does -6? **Yes** Does 0? **No** Does 1? **No** Does -1? **No** Does 8? **No** Does -8? **Yes**

Page 8

Does $-5\frac{1}{2}$ belong to this set? **No** Does -6? **Yes** Does 0? **Yes** Does 1? **Yes** Does -1? **Yes** Does 8? **No** Does -8? **No**

Page 9

How many points are there in the graph of set A? **5**
Are there an infinite number of points in the graph of set B? **Yes**
Does -1 belong to set B? **No** Does -12? **Yes** Does 5? **No** Does 0? **No** Does -2? **Yes** Does -8? **Yes** Does -20? **Yes**

Page 10 **Exercise – The Set of Integers**

1. (a) right
   (b) 0
   (c) -4
   (d) greater
   (e) right
   (f) negative
   (g) magnitude
   (h) $|a + b|$
   (i) magnitude, directions (or signs)
   (j) -5
   (k) greater
   (l) positive
   (m) natural

2. (b) $-7, -6, -4, 0, 2, 5$
   (c) $-20, -10, -5, 0, 5, 10, 20$
   (d) $-16, -12, -8, -4, 2, 6, 10, 14$
   (e) $-8, -5, -3, -1, 0, 1, 6$

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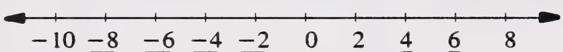
3. (a) true
   (b) false
   (c) true
   (d) false
   (e) true
   (f) false
   (g) false
   (h) true

4. (b) -9
   (c) $-9, -8, -7$
   (d) $-2, -1, 0$
   (e) $-5, -4, -3$
   (f) -1
   (g) $-8, -7, -6, -5$ (list any 3 of these)

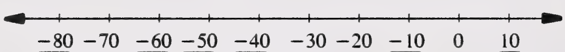
5. (b) $\{-1, -2, -3, \ldots\}$
   (c) $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
   (d) $\{0, 1, 2, 3, \ldots\}$
   (e) $\{0, -1, -2, -3, \ldots\}$
   (f) $\{-1, 0, 1, 2, \ldots\}$
   (g) $\{-3, -2, -1, 0, 1, 2\}$

6. (a)

   (b)



   (c)



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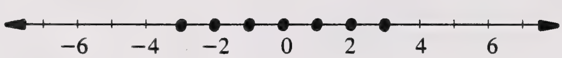
7. (a) $<$, 13 is to the left of 17
   (b) $>$, 10 is to the right of 0
   (c) $>$, 10 is to the right of -10
   (d) $<$, -10 is to the left of 0
   (e) $>$, -10 is to the right of -20
   (f) $<$, -13 is to the left of -12
   (g) $<$, -5 is to the left of 0
   (h) $>$, -3 is to the right of -6

8. (b) $\{x \mid -4 < x < 4, x \in I\}$
   $\{-3, -2, -1, 0, 1, 2, 3\}$



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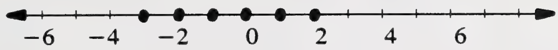
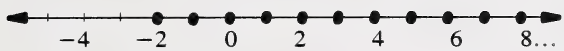
(c) $\{x \mid -4 < x \leq 2, x \in I\}$
$\{-3, -2, -1, 0, 1, 2\}$



(d) $\{x \mid x > -3, x \in I\}$
$\{-2, -1, 0, 1, 2, 3, \ldots\}$



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The integer +2 represents a trip of 2 units to the **right** of the origin. On the number line above, the integer +2 is represented by a trip from point **D** to point F. From point F, you must move **5** units to the **right** to represent the integer +5. In the second trip, you go from point F to point **K**. Thus, the sum (+2) + (+5) could also be represented by a single trip of +7.

(+4) + (+2) = **+6**
(+2) + (+4) = **+6**

(+7) + (+9) = **+16**
(+15) + (+3) = **+18**

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(b) (+9) + (+3) = **+12**
(c) (+6) + (+3) = **+9**
(d) (+100) + (+5) = **+105**

The integer −2 represents a trip of **2** units to the **left** of the origin. On the number line above, you move from point **I** to point **G**. The integer −5 represents a trip of **5** units to the **left.**

On the number line, you move from point G to point **B.**

(−4) + (−2) = **−6**
(−2) + (−4) = **−6**

(−7) + (−9) = **−16**
(−15) + (−3) = **−18**

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(b) (−9) + (−3) = **−12**
(c) (−6) + (−3) = **−9**
(d) (−100) + (−5) = **−105**

The integer +5 represents a trip of **5** units to the **right**. On the number line above, you move from point **G** to point **L.** From this point, you go **3** units to the **left** to represent the integer −3. In the second trip you go from point L to point **I.**

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$(+8) + (-5) = \mathbf{+3}$
$(-8) + (+5) = \mathbf{-3}$

$(+3) + (-7) = \mathbf{-4}$
$(-3) + (+7) = \mathbf{+4}$

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(b) $(+8) + (-2) = \mathbf{+6}$
(c) $(-9) + (+1) = \mathbf{-8}$
(d) $(-9) + (+10) = \mathbf{+1}$

Page 23 **Exercise – Adding Integers**

1. (a) positive, adding
   (b) negative, adding the absolute values of the two numbers
   (c) (i) difference
       (ii) positive
       (iii) negative

2. (a) -7
   (b) $+7$
   (c) $+23$
   (d) -23
   (e) 0
   (f) $+5$
   (g) -6
   (h) $+12$
   (i) -19
   (j) $+1$
   (k) 0
   (l) -1
   (m) $(-9) + (+3) = -6$
   (n) $-1 + (-4) = -5$
   (o) $(-3) + (+15) = +12$
   (p) $(+6) + (-6) = 0$
   (q) $0 + (-7) = -7$
   (r) $(+11) + (-12) = -1$

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What is the additive inverse of -4? **4** of 6? $\mathbf{-6}$ of x? $\mathbf{-x}$

1. $(-2) + (+2) = \mathbf{0}$
2. $\mathbf{(+3)} + (-3) = 0$
3. $(+7) + (-7) = 0$
4. $(-14) + \mathbf{(+14)} = 0$
5. $(+6) + (-6) = \mathbf{0}$
6. $x + \mathbf{(-x)} = 0$

Page 28 **Exercise – Subtracting Integers**

1. (a) 0, additive inverse
   (b) negative, same
   (c) adding, b
   (d) $(-2) + (+5)$, different

2. (b) $(-18) + (-13) = -31$
   (c) $(-6) + (+8) = +2$
   (d) $(-9) + (+9) = 0$
   (e) $(+6) + (-14) = -8$
   (f) $(+12) + (-3) = +9$
   (h) $[(+4) + (-1)] + (+8) = (+3) + (+8) = +11$
   (i) $[(+6) + (+2)] + (-8) = (+8) + (-8) = 0$

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(b) $+30$
(c) $+9$
(d) $+100$

Page 31

(b) $+21$
(c) $+81$
(d) $+12$

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$(-2) \times (+11) = -22$
$(+8) \times (-3) = -24$
$(+9) \times (-9) = -81$

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$(+8) \div (+2) = +4$
$(-16) \div (+4) = -4$
$(+24) \div (-6) = -4$
$(-12) \div (-4) = +3$
$(-32) \div (+8) = -4$
$(-4) \div (-1) = +4$

Page 36 **Exercise – Multiplying and Dividing Integers**

1. (a) (i) postive, negative
   (ii) negative
   (iii) product, absolute

   (b) positive
   (c) positive

2. (a) -18
   (b) $+52$
   (c) -7
   (d) -11
   (e) $+34$
   (f) $+24$
   (g) $+3$
   (h) -75
   (i) $+1$
   (j) -48
   (k) $(+40) \div (+2) = +20$
   (l) $(-4) \times (-6) = +24$
   (m) $(+8) \div (-2) = -4$

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How would you write $(+12) \div (+3)$ in simplier form? $\mathbf{12 \div 3}$

3. $(-8) \div 2$ represents the quotient of **negative** 8 and **positive** 2. Since the signs are **different**, the quotient will be **negative** $(-8) \div 2 = \mathbf{-4}$

4. $3 + (-6)$ represents the sum of **positive** 3 and **negative** 6. In order to find the sum, we must **subtract** the two absolute values and attach the sign of the number with the **larger** absolute value.

   $3 + (-6) = \mathbf{-3}$

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1. 10
2. 1
3. 5
4. -5

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3. negative, positive
   negative
   -11

Page 41 **Exercise – Operating with Integers**

1. (a) 3
   (b) -4
   (c) 1
   (d) 0
   (e) 2
   (f) -28
   (g) -23
   (h) 28
   (i) $4 + (-14) = -10$
   (j) $(-4) + (-14) = -18$
   (k) 10
   (l) $3 + (-7) = -4$
   (m) 4
   (n) -13
   (o) $8 + (2) = 10$
   (p) $(-12) + (+7) = -5$
   (q) $6 + (-9) = -3$
   (r) $(-6) + (+8) = +2$
   (s) -72
   (t) $+16$
   (u) -6
   (v) -2
   (w) -5
   (x) $+2$
   (y) 120
   (z) $+16$

2. (a) $= 18 - (-28)$
   $= 18 + (28)$
   $= 46$

   (b) $= 4\,[(-6) + (+3)]$
   $= 4 \times (-3)$
   $= -12$

   (c) $=(-15) - 4$
   $= (-15) + (-4)$
   $= -19$

   (d) $= 32 \div -2$
   $= -16$

(e) $= 6 - 20$
$= 6 + (-20)$
$= -14$

(f) $= 3 + [(-5) + (-1)]$
$= 3 + (-6)$
$= -3$

(g) $4 \times [3 + (-8)]$
$= 4 \times (-5)$
$= -20$

(h) $= (-2 + 5) - 8$
$= 3 + (-8)$
$= -5$

2. -18
3. $-3, 3$
4. -4

Page 43

1. -2
2. -24
3. $-8, 0$
4. $1, 4$

Left side $= -3(-2 + 6)$
$= -3 \times \mathbf{4}$
$= \mathbf{-12}$

Right side $= (-3 \times -2) + (-3 \times 6)$
$= \mathbf{6 + (-18)}$
$= \mathbf{-12}$

Are the two sides equal? **Yes**

Left side $= -3(5-9)$
$= -3 \times -4$
$= +12$

Right side $= (-3 \times 5) - (-3 \times 9)$
$= -15 - (-27)$
$= -15 + 27$
$= +12$

Are the two sides equal? **Yes**

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$-5 + 2 = -3$
$-4 \times -2 = 8$

$6 - 3 = 3$
$7 - 9 = -2$

$9 \times -7 = -63$
$(-3) \times (-4) = 12$

$4 \div 7$ **N**
$8 \div -2$
$-2 \div 8$ N

$-16 \div -4$
$-14 \div -16$ N
$14 \div 5$ **N**

$-8 + 0 = \mathbf{-8}$
$\mathbf{0} + (-3) = -3$
$0 + \mathbf{6} = 6$

$1 \times -4 = \mathbf{-4}$
$\mathbf{1} \times 3 = 3$
$\mathbf{-6} \times 1 = -6$

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The additive inverse of 4 is $\mathbf{-4}$ since $4 + \mathbf{-4} = 0$
The additive inverse of -8 is $\mathbf{8}$ since $\mathbf{-8} + \mathbf{8} = \mathbf{0}$.

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1. (a) addition, multiplication, division
   (b) subtraction
   (c) additive indentity
   (d) multiplicative identity
   (e) addition, multiplication, subtraction, division
   (f) addition, subtraction

2. (b) Commutative property of multiplication
   (c) Commutative property of addition
   (d) Closure property of addition
   (e) Associative property of addition
   (f) Distributive property
   (g) Closure property of multiplication
   (h) Additive inverse property
   (i) Multiplicative identity property
   (j) Commutative property of multiplication
   (k) Additive identity property
   (l) Associative property of multiplication

END OF LESSON 3

LESSON 4

Page 1

1. $-16 \div -4$ **I**
2. $8 \div -3$
3. $-3 \div 7$
4. $0 \div 5$ **I**
5. $20 \div -5$ **I**
6. $-5 \div -5$ **I**
7. $6 \div 4$
8. $-2 \div 6$

Page 2

Give examples of other fractions that do not exist because of this reason.

$\frac{1}{0}$ $\frac{6}{0}$ $\frac{-7}{0}$ or any other fraction that has zero as the denominator

1. Numerator is 6. Denominator is -11.
2. Numerator is -7. Denominator is -9.
3. Numerator is 0. Denominator is 8.

Page 3

1. $\frac{4}{3}$
2. $\frac{12}{-4}$ I
3. $\frac{0}{-11}$ **I**
4. $\frac{-15}{-3}$ **I**
5. $\frac{-2}{4}$
6. $\frac{18}{2}$ **I**

1. $\mathbf{\frac{5}{-5} = -1}$
2. $\frac{-33}{-3} = \mathbf{11}$
3. $\frac{14}{2} = \mathbf{7}$
4. $\frac{0}{2} = \mathbf{0}$
5. $\frac{-24}{6} = \mathbf{-4}$
6. $\frac{-7}{-7} = \mathbf{1}$

1. $1, \mathbf{\frac{8}{8}, \frac{-2}{-2}, \frac{1}{1}, \frac{4}{4}}$
2. $-5, \mathbf{\frac{-35}{7}, \frac{-5}{1}, \frac{-10}{2}, \frac{30}{-6}}$
3. $-1, \mathbf{\frac{-12}{12}, \frac{-1}{+1}, \frac{6}{-6}, \frac{5}{-5}}$
4. $12, \mathbf{\frac{-12}{-1}, \frac{12}{1}, \frac{-24}{-2}, \frac{48}{4}}$
5. $0, \mathbf{\frac{0}{3}, \frac{0}{6}, \frac{0}{-7}, \frac{0}{1}}$
6. $4, \mathbf{\frac{36}{9}, \frac{4}{1}, \frac{-8}{-2}, \frac{12}{3}}$

(For numbers 1 to 6 you can have any other similar answer.)

Page 4

Is -6 a rational number? **Yes** Is $\frac{1}{2}$? **Yes** Is 10? **Yes** Is $\frac{-2}{-3}$? **Yes** Is $\frac{5}{2}$? **Yes**

Is $\frac{3}{0}$? **No** Is $\frac{0}{3}$? Yes

Page 5

How would you write $\frac{-1}{-5}$? $\mathbf{\frac{1}{5}}$ $\frac{-3}{-5}$? $\mathbf{\frac{3}{5}}$ $\frac{-6}{-5}$? $\mathbf{\frac{6}{5}}$ $\frac{2}{3}$? $\mathbf{\frac{2}{3}}$ $\frac{-2}{-3}$? $\mathbf{\frac{2}{3}}$

How would you write $\frac{5}{-8}$? $\mathbf{\frac{5}{-8}}$ $\frac{-5}{8}$? $\mathbf{\frac{-5}{8}}$ $\frac{10}{-11}$? $\mathbf{\frac{-10}{11}}$ $\frac{4}{-7}$? $\mathbf{\frac{-4}{7}}$

$\frac{-1}{3}$? $\mathbf{\frac{-1}{3}}$ $\frac{1}{-3}$? $\mathbf{\frac{-1}{3}}$

Page 7

Fraction Pair	Cross Products	Equal or Not Equal
$\frac{6}{8}, \frac{4}{6}$	$6 \times 6 = 36$ / $8 \times 4 = 32$	not equal
$\frac{-20}{16}, \frac{-5}{4}$	$-20 \times 4 = -80$ / $16 \times -5 = -80$	equal
$\frac{0}{5}, \frac{0}{7}$	$0 \times 7 = 0$ / $0 \times 5 = 0$	equal
$\frac{6}{8}, \frac{-3}{2}$	$6 \times 2 = 12$ / $8 \times -3 = -24$	not equal

Page 8

1. $\frac{3}{\boxed{6}}$
2. $\frac{7}{\boxed{4}}$
3. $\frac{-1}{\boxed{2}}$
4. $\frac{-3}{\boxed{4}}$
5. $\frac{6}{\boxed{14}}$
6. $\frac{4}{\boxed{5}}$

Give four rational numbers that are equivalent to the given number.

1. $\frac{-1}{4}, \frac{-2}{8}, \frac{-3}{12}, \frac{-4}{16}, \frac{-5}{20}$
2. $\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}$
3. $\frac{4}{3}, \frac{8}{6}, \frac{12}{9}, \frac{16}{12}, \frac{20}{15}$
4. $\frac{-15}{60}, \frac{-5}{20}, \frac{-1}{4}, \frac{-2}{8}, \frac{-3}{12}$

(For questions 1 to 4 you can have any other similar answers.)

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2. $\frac{32}{50}$
3. $\frac{-6}{21}$
4. $\frac{-28}{35}$
5. $\frac{6}{11}$
6. $\frac{-6}{-10}$

1. $\frac{-6}{14}$
2. $\frac{15}{10}$
3. $\frac{20}{24}$
4. $\frac{12}{3}$
5. $\frac{-6}{15}$
6. $\frac{8}{16}$

Page 10

1. $\frac{-6}{12}$
2. $\frac{6}{4}$
3. $\frac{3}{2}$
4. $\frac{-3}{14}$
5. $\frac{9}{1}$
6. $\frac{-3}{4}$

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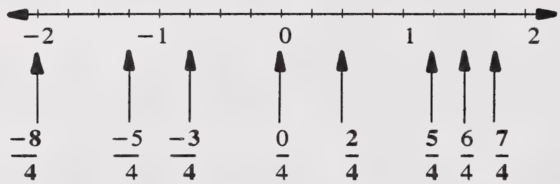
How many integers are there between 3 and 5? **one**

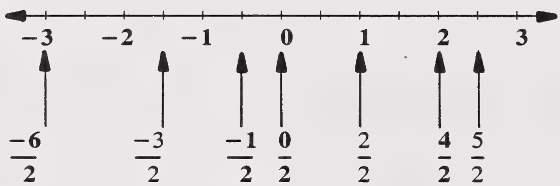
Between -1 and -4? **2** Between 2 and 3? **none**

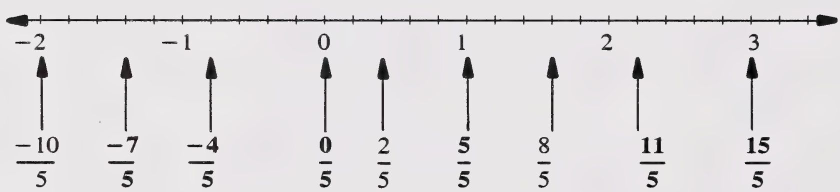
Between -8 and -9? **none**

Between -1 and -2? **infinite number**

Between $\frac{1}{3}$ and $\frac{2}{3}$? **infinite number**







1. $\frac{5}{8}$ ✓
2. $\frac{8}{7}$ ____
3. $\frac{-1}{2}$ ____
4. $\frac{4}{8}$ ✓
5. $\frac{16}{17}$ ✓
6. $\frac{-6}{13}$ ____

1. $\frac{-3}{4}$ ✓
2. $\frac{-16}{12}$ ____
3. $\frac{10}{3}$ ____
4. $\frac{-1}{6}$ ✓
5. $\frac{-7}{9}$ ✓
6. $\frac{-14}{2}$ ____

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2. $>$, right
3. $<$, left
4. $>$, right
5. $>$, right
6. $<$, left
7. $>$, right

Page 19

1. $[(3 \times 5) + 2]$ fifths $= \dfrac{17}{5}$

2. $[(7 + 8) + 1]$ eighths $= \dfrac{57}{8}$

3. $[(5 \times 7) + 4]$ sevenths $= \dfrac{39}{7}$

4. $[(2 \times 9) + 5]$ ninths $= \dfrac{23}{9}$

5. $[(7 \times 3) + 2]$ thirds $= \dfrac{23}{3}$

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1. $-[(4 \times 5) + 3]$ fifths $= \dfrac{-23}{5}$

2. $-[(5 \times 2) + 1]$ halves $= \dfrac{-11}{2}$

3. $-[(1 \times 10) + 9]$ tenths $= \dfrac{-19}{10}$

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	Required Division	Mixed Number
1.	$21 \div 8 = 2$, remainder 5	$-2\frac{5}{8}$
2.	$35 \div 6 = 5$, remainder 5	$5\frac{5}{6}$
3.	$11 \div 10 = 1$, remainder 1	$1\frac{1}{10}$
4.	$17 \div 2 = 8$, remainder 1	$-8\frac{1}{2}$
5.	$63 \div 5 = 12$, remainder 3	$12\frac{3}{5}$

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1. (a) numerator, denominator, terms
   (b) np
   (c) prime
   (d) smaller
   (e) less, ed
   (f) dense
   (g) 2, 5, 7
   (h) multiplied, divided
   (i) greater, less
   (j) zero
   (k) dividing, 8
   (l) integer
   (m) $-5 \times \mathbf{15} \div 8 \times \mathbf{-9}$
   (n) mixed
   (o) highest common
   (p) right
   (q) subset

2. (a) $\frac{35}{7}$ (b) $\frac{-3}{1}$

   (c) $\frac{2}{1}$ (d) $\frac{0}{1}$

   (e) $\frac{-11}{1}$ (f) $\frac{6}{1}$

3. (b)
$$\begin{aligned} 490 &= 2 \times 5 \times 7 \times 7 \\ 245 &= 5 \times 7 \times 7 \\ \text{H.C.F.} &= 5 \times 7 \times 7 \\ &= 245 \end{aligned}$$

   (c)
$$\begin{aligned} 144 &= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \\ 108 &= 2 \times 2 \times 3 \times 3 \times 3 \\ \text{H.C.F.} &= 2 \times 2 \times 3 \times 3 \\ &= 36 \end{aligned}$$

   (d)
$$\begin{aligned} 75 &= 3 \times 5 \times 5 \\ 135 &= 3 \times 3 \times 3 \times 5 \\ \text{H.C.F.} &= 3 \times 5 \\ &= 15 \end{aligned}$$

4.

Fraction	Step 1	Step 2	Step 3
$\frac{30}{50}$	$\frac{30}{50} = \frac{10 \times 3}{10 \times 5}$	H.C.F. = 10	$\frac{30 \div 10}{50 \div 10} = \frac{3}{5}$
$\frac{-75}{225}$	$\frac{-75}{225} = \frac{75 \times -1}{75 \times 3}$	H.C.F. = 75	$\frac{-75 \div 75}{225 \div 75} = \frac{-1}{3}$
$\frac{48}{168}$	$\frac{48}{168} = \frac{24 \times 2}{24 \times 7}$	H.C.F. = 24	$\frac{48 \div 24}{168 \div 24} = \frac{2}{7}$
$\frac{-70}{105}$	$\frac{-70}{105} = \frac{35 \times -2}{35 \times 3}$	H.C.F. = 35	$\frac{-70 \div 35}{105 \div 35} = \frac{-2}{3}$

5.

Circle equivalent Forms						
$\frac{-2}{-8}$,	$\frac{2}{-8}$,	$\frac{-2}{8}$,	$\frac{-1}{-4}$,	$\frac{8}{24}$,	$\frac{1}{4}$,	$\frac{-20}{80}$
$\frac{4}{5}$,	$\frac{4}{-5}$,	$\frac{-4}{-5}$,	$\frac{12}{15}$,	$\frac{-8}{10}$,	$\frac{-20}{25}$,	$\frac{-12}{10}$
$\frac{-36}{3}$,	$\frac{36}{-3}$,	$\frac{36}{3}$,	$\frac{-12}{-1}$,	$\frac{12}{-1}$,	$\frac{-12}{1}$	$\frac{60}{-5}$

6.

Cross Products		Ordering of Fractions
ad	bc	
$-2 \times 15 = -30$	$5 \times -16 = -80$	$\frac{-2}{5} > \frac{-16}{15}$
$9 \times 30 = 270$	$25 \times 11 = 275$	$\frac{9}{25} < \frac{11}{30}$
$-8 \times 18 = -144$	$3 \times -48 = -144$	$\frac{-8}{3} = \frac{-48}{18}$
$-21 \times 12 = -252$	$36 \times -7 = -252$	$\frac{-21}{36} = \frac{-7}{12}$

7. (b) $\frac{84}{36} = \frac{84 \div 12}{36 \div 12} = \frac{7}{3} = 2\frac{1}{3}$

(c) $\frac{-54}{16} = \frac{-54 \div 2}{16 \div 2} = \frac{-27}{8} = -3\frac{3}{8}$

(d) $\frac{-90}{72} = \frac{-90 \div 18}{72 \div 18} = \frac{-5}{4} = -1\frac{1}{4}$

8.

Rational Number	Mixed Number	Position on Number Line
$\frac{13}{2}$	$6\frac{1}{2}$	Lies between 6 and 7
$\frac{-65}{12}$	$-5\frac{5}{12}$	Lies between -6 and -5
$\frac{14}{9}$	$1\frac{5}{9}$	Lies bewtween 1 and 2
$\frac{-19}{8}$	$-2\frac{3}{8}$	Lies between -2 and -3
$\frac{99}{41}$	$2\frac{17}{41}$	Lies between 2 and 3

9. (a) 0 and 1
   (b) 0 and -1

10. (a) $<$
    (b) $=$
    (c) $>$
    (d) $=$

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$$\frac{-1}{4} \times \frac{-3}{2} = \frac{3}{8}$$

$$\frac{-1}{8} \times \frac{7}{9} = \frac{-7}{72}$$

$$\frac{-3}{4} \times \frac{1}{2} = \frac{-3}{8}$$

2. $\frac{-20}{80} = \frac{-1}{4}$

3. $\frac{-2 \times -3}{5 \times 16} = \frac{6}{80} = \frac{3}{40}$

4. $\frac{5 \times -18}{9 \times 25} = \frac{-90}{225} = \frac{-2}{5}$

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3. $\frac{5}{4}$

4. $\frac{3}{-7}$

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$\frac{3}{8} + \frac{2}{8} = \mathbf{\frac{5}{8}}$

$\frac{1}{6} + \frac{4}{6} = \mathbf{\frac{5}{6}}$

$\frac{5}{12} - \frac{4}{12} = \mathbf{\frac{1}{12}}$

$\frac{4}{7} - \frac{1}{7} = \mathbf{\frac{3}{7}}$

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2. $= \frac{10}{10} = 1$

3. $\frac{4}{9}$

4. $\frac{-2}{3}$

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$\frac{5}{3} \times \frac{2}{7} = \mathbf{\frac{10}{21}}$

$= \frac{5}{6} \times \frac{1}{5} = \mathbf{\frac{1}{6}}$

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Exercise – Operating with Rational Numbers

1. (a) m + n

   (b) m − n

   (c) ac

   (d) ad

   (e) $\frac{3}{2}$

   (f) multiplying $\frac{4}{3}$

   (g) least common

   (h) reciprocal

(i) 180

(j) numerator, denominator

(k) product, denominator

(l) $\frac{-25}{8}, \frac{19}{7}$

2. (b) $\frac{-3 + 5}{16} = \frac{2}{16} = \frac{1}{8}$

(c) $\frac{-7 + -17}{18} = \frac{-24}{18} = \frac{-4}{3} = -1\frac{1}{3}$

(d) $\frac{11 + 12}{15} = \frac{23}{15} = 1\frac{8}{15}$

(e) $\frac{-26}{12} = \frac{-13}{6} = -2\frac{1}{6}$

(f) $\frac{-4}{3} = -1\frac{1}{3}$

(g) $\frac{28}{18} = \frac{14}{9} = 1\frac{5}{9}$

(h) $\frac{-3 + 3}{4} = \frac{0}{4} = 0$

(j) $\frac{\cancel{-11}^{\,-1}}{\cancel{5}_{\,1}} \times \frac{\cancel{5}^{\,1}}{\cancel{11}_{\,1}} = \frac{-1 \times 1}{1 \times 1} = \frac{-1}{1} = -1$

(k) $\frac{\cancel{93}^{\,3}}{{}_1\cancel{3}} \times \frac{\cancel{-3}^{\,-1}}{\cancel{31}_{\,1}} = \frac{3 \times -1}{1 \times 1} = \frac{-3}{1} = -3$

(l) $\cancel{36}^{\,6} \times \frac{11}{\cancel{6}_{\,1}} = \frac{6 \times 11}{1} = \frac{66}{1} = 66$

(m) $\frac{-23}{\cancel{9}_{\,1}} \times \frac{\cancel{9}^{\,1}}{4} = \frac{-23 \times 1}{1 \times 4} = \frac{-23}{4} = -5\frac{3}{4}$

(n) $\frac{\cancel{26}^{\,13}}{\cancel{9}_{\,1}} \times \frac{\cancel{27}^{\,3}}{\cancel{2}_{\,1}} = \frac{13 \times 3}{1 \times 1} = \frac{39}{1} = 39$

(o) $\dfrac{\overset{-1}{\cancel{-7}} \times \overset{2}{\cancel{10}}}{\underset{1}{\cancel{5}} \times \underset{3}{\cancel{21}}} = \dfrac{-1 \times 2}{1 \times 3} = \dfrac{-2}{3}$

(p) $\dfrac{8}{9} \times \dfrac{1}{3} = \dfrac{8}{27}$

(q) $\overset{5}{\cancel{-25}} \times \dfrac{3}{\underset{-2}{\cancel{-10}}} = \dfrac{5 \times 3}{2} = \dfrac{15}{2} = 7\dfrac{1}{2}$

(r) $\dfrac{-15}{2} \div \dfrac{25}{4} = \dfrac{\overset{-3}{\cancel{-15}}}{\underset{1}{\cancel{2}}} \times \dfrac{\overset{2}{\cancel{4}}}{\underset{5}{\cancel{25}}} = \dfrac{-3 \times 2}{1 \times 5} = \dfrac{-6}{5} = -1\dfrac{1}{5}$

(s) $\dfrac{32}{9} \div \dfrac{16}{5} = \dfrac{\overset{2}{\cancel{32}}}{9} \times \dfrac{5}{\underset{1}{\cancel{16}}} = \dfrac{2 \times 5}{9 \times 1} = \dfrac{10}{9} = 1\dfrac{1}{9}$

(t) $\dfrac{8}{18} = \dfrac{4}{9}$

(u) $\dfrac{5 \times 5}{36 \times 5} + \dfrac{7 \times 6}{30 \times 6} = \dfrac{25 + 42}{180} = \dfrac{67}{180}$

(v) $\dfrac{1 \times 2}{60 \times 2} - \dfrac{5}{120} = \dfrac{2 - 5}{120} = \dfrac{-3}{120} = \dfrac{-1}{40}$

(w) $\dfrac{-5 \times 3}{36 \times 3} + \dfrac{7 \times 2}{54 \times 2} = \dfrac{-15 + 14}{108} = \dfrac{-1}{108}$

(x) $\dfrac{-5 \times 3}{6 \times 3} + \dfrac{-5 \times 2}{9 \times 2} = \dfrac{-15 + -10}{18} = \dfrac{-25}{18} = -1\dfrac{7}{18}$

3. (a) -5

(b) $\dfrac{9}{4} \div \dfrac{3}{1} = \dfrac{\overset{3}{\cancel{9}}}{4} \times \dfrac{1}{\underset{1}{\cancel{3}}} = \dfrac{3 \times 1}{4 \times 1} = \dfrac{3}{4}$

(c) $\dfrac{5}{8} \div \dfrac{15}{4} = \dfrac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{8}}} \times \dfrac{\overset{1}{\cancel{4}}}{\underset{3}{\cancel{15}}} = \dfrac{1 \times 1}{2 \times 3} = \dfrac{1}{6}$

(d) $\left(\cancel{4}^{2} \times \frac{3}{\cancel{2}_{1}}\right) \div \frac{1}{2} = \frac{2 \times 3}{1} \times \frac{2}{1} = 6 \times 2 = 12$

(e) $\left(\frac{2 \times 4}{3 \times 4} + \frac{1 \times 3}{4 \times 3}\right) \div 4 = \left(\frac{8 + 3}{12}\right) \times \frac{1}{4} = \frac{11}{12} \times \frac{1}{4} = \frac{11}{48}$

(f) $\frac{1}{4} \div \left(\frac{7}{8} - \frac{1}{8}\right) = \frac{1}{4} \div \frac{6}{8} = \frac{1}{\cancel{4}_{1}} \times \frac{\cancel{8}^{2}}{6} = \frac{2}{6} = \frac{1}{3}$

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1. $\frac{\cancel{-3}^{-1}}{\cancel{2}_{1}} \times \frac{\cancel{4}^{2}}{\cancel{9}_{3}} = \frac{-1 \times 2}{1 \times 3} = \frac{-2}{3}$

2. $\cancel{6}^{1} \times \frac{13}{\cancel{12}_{2}} = \frac{1 \times 13}{2} = \frac{13}{2} = 6\frac{1}{2}$

3. $\frac{0}{5} \times \frac{3}{8} = \frac{0}{40} = 0$

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1. $\frac{1}{2} + \frac{0}{2} = \mathbf{\frac{1}{2}} \qquad 0 + \frac{3}{4} = \mathbf{\frac{3}{4}} \qquad 0 + \mathbf{\frac{-3}{8}} = \frac{-3}{8}$

2. $\frac{-6}{7} \times 1 = \frac{-6}{7} \qquad \frac{3}{4} \times \frac{2}{2} = \mathbf{\frac{3}{4}} \qquad \frac{-4}{3} \times \mathbf{\frac{6}{6}} = \frac{-4}{3}$

3 and -3 are additive inverses since $3 + (-3) = \mathbf{0}$.
-5 and **5** are additive inverses since $-5 + \mathbf{5} = 0$.
0 and **0** are additive inverses since $0 + \mathbf{0} = \mathbf{0}$

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$\frac{2}{3}$ and $\frac{-2}{3}$ are additive inverses since $\frac{2}{3} + \mathbf{\frac{-2}{3}} = 0$

$\frac{-5}{7} + \frac{5}{7}$ are additive inverses since $\frac{-5}{7} + \mathbf{\frac{5}{7}} = 0$

$\frac{1}{4} + \mathbf{\frac{-1}{4}}$ are additive inverses since $\frac{1}{4} + \mathbf{\frac{-1}{4}} = \mathbf{0}$

$\frac{-2}{3}$ and $\frac{3}{-2}$ are multiplicative inverses since $\frac{-2}{3} \times \frac{3}{-2} = 1$

$\frac{1}{2}$ and 2 are multiplicative inverses since $\frac{1}{2} \times \frac{2}{1} = 1$

$\frac{4}{3}$ and $\frac{3}{4}$ are multiplicative inverses since $\frac{4}{3} \times \frac{3}{4} = 1$

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Addition	Multiplication
$\left(\frac{a}{b} + \frac{c}{d}\right)$ is a rational number	$\left(\frac{a}{b} \times \frac{c}{d}\right)$ is a **rational** number
$\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$	$\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$
$\frac{a}{b} + \left(\frac{c}{d} + \frac{e}{f}\right) = \left(\frac{a}{b} + \frac{c}{d}\right) + \frac{e}{f}$	$\frac{a}{b}\left(\frac{c}{d} \times \frac{e}{f}\right) = \left(\frac{a}{b} \times \frac{c}{d}\right) \frac{e}{f}$
$\frac{a}{b}\left(\frac{c}{d} + \frac{e}{f}\right) = \left(\frac{a}{b} \times \frac{c}{d}\right) + \left(\frac{a}{b} + \frac{e}{f}\right)$	
Additive identity is zero. $\frac{a}{b} + 0 = \frac{a}{b}$	Multiplicative identity is one. $\frac{a}{b} \times 1 = \frac{a}{b}$
Additive inverse of $\frac{a}{b}$ **is** $\frac{-a}{b}$; $\frac{a}{b} + \frac{-a}{b} = 0$	**Multplicative inverse of** $\frac{a}{b}$ **is** $\frac{b}{a}$; $\frac{a}{b} \times \frac{b}{a} = 1$

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Exercise – Properties of Set Q

1. (a) additive
   (b) zero
   (c) addition, subtraction
   (d) multiplicative inverse
   (e) subtraction, division
   (f) multiplicative
   (g) zero
   (h) zero
   (i) additive .

2. (b) $\frac{-6}{11}, \frac{11}{6}$

   (d) $\frac{-17}{4}, \frac{4}{17}$

   (e) $-3, \frac{1}{3}$

   (f) $\frac{1}{7}, \frac{7}{-1}$

   (g) $\frac{-y}{x + z}, \frac{x + z}{y}$

3.

(b)	Division is not commutative	false
(c)	0 is the additive identity	true
(d)	Subtraction is not commutative	false
(e)	Addition is associative	true
(f)	Addition has inverse elements	true
(g)	Set Q is closed under addition	true
(h)	Division is not associative	false
(i)	1 is the multiplicative identity	true
(j)	Multiplication distributes over subtraction	true
(k)	Multiplication has inverse elements	true
(l)	Addition is commutative	true

END OF LESSON 4

LESSON 5

Page 1

8 643 970 042	123 478 900 462
46 322 875	8 933 607
66 372 489 832	235 562 831

Page 2

1. thousands period? **302** In the millions period? **506**

2. **150 200 000 097**

   billions period? **150** thousands period? **000**
   "150 **billion,** 200 **million**, 97."

3. **35 072 903**

   billions period? **none** thousands period? **072** million period? **35**
   "35 **million**, 72 **thousand**, 903."

Page 3

1. (ii) billions, 1 000 000 000
   (iii) ten millions, ($6 \times 10\ 000\ 000$) or 60 000 000

2. 23 009 674

   (i) 3, ($3 \times 1\ 000\ 000$) or 3 000 000
   (ii) 0, ($0 \times 100\ 000$)
   (iii) 9, (9×1000) or 9000

3. 560 007

   (i) hundred thousands, ($5 \times 100\ 000$) or 500 000
   (ii) 0, (0×1000) or 0.
   (iii) units, (7×1) or 7.

Page 6

3 **thousands** or (3 × **1000**)
2 **hundreds** or (2 × 100)
0 **tens** or (0 × **10**)
9 one or (9 × **1**)

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(ii) hundredths
(iii) thousands, thousandths

1. (ii) thousands
   (iii) hundreds, (3 × 100) or 300
   (iv) tens, (4 × 10) or 40
   (v) units, (5 × 1) or 5
   (vi) tenths
   (vii) hundredths, $\frac{7}{100}$

2. 62.008
   (i) 6, (6 × 10) or 60
   (ii) 2, (2 × 1) or 2
   (iii) 8, $(8 \times \frac{1}{1000})$ or $\frac{8}{1000}$
   (iv) hundredths
   (v) units, tenths

3. 3 041.2579
   (i) hundreds place? 0 In the thousandths place? 7 In the ten-thousandths place? 9 In the thousands place? 3 In the tenths place? 2 In the tens place? 4
   (ii) hundredths, $(5 \times \frac{1}{100})$ or $\frac{5}{100}$
   (iii) 1, 2

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7.165 is read "seven, **decimal** one, six, **five**"
60.04 is read "60, decimal **zero** four"
235.92 is read "two **hundred** thirty-five, decimal **nine, two**.

5.832
3006.042
70.6
300.0186

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6 ones or (**6** $\times$ 1)

3 tenths or (**3** $\times \frac{1}{10}$)

0 **hundredths** or (0 $\times \frac{1}{100}$)

7 **thousandths** or 7 $\times \frac{\mathbf{1}}{\mathbf{1000}}$

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Exercise – Decimal Numbers

1. (a) ten
   (b) place
   (c) millions
   (d) billion, thousand
   (e) left
   (f) right
   (g) ten-thousands
   (h) ths
   (i) three, left, two, right
   (j) one hundred twenty, decimal, two, six, three
   (k) hundredths

2. (c) hundreds, 600
   (d) hundredths, $\frac{7}{100}$
   (e) thousands, 1000
   (f) thousandths, $\frac{8}{1000}$
   (g) units, 7
   (h) ten thousandths, $\frac{2}{10\,000}$
   (i) ten thousands, 30 000
   (j) tenths, $\frac{9}{10}$

3. (b) (i) thirteen, decimal, five, one

(ii) $(1 \times 10) + (3 \times 1) + (5 \times \frac{1}{10}) + (1 \times \frac{1}{100})$

(c) (i) one, decimal, zero, zero, three, five

(ii) $(1 \times 1) + (0 \times \frac{1}{10}) + (0 + \frac{1}{100}) + (3 \times \frac{1}{1000}) + (5 \times \frac{1}{10\ 000})$

(d) (i) sixty thousand one hundred, decimal, eight, two

(ii) $(6 \times 10\ 000) + (0 \times 1000) + (1 \times 100) + (0 \times 10) + (0 \times 1) + (8 \times \frac{1}{10}$ $+ (2 \times \frac{1}{100})$

Page 20

4. (a) <
(b) <
(c) =
(d) <
(e) >
(f) >
(g) >
(h) =
(i) <
(j) <
(k) >
(l) <

5. (b) 0.07, 0.077, 0.691, 0.7
(c) 1.29, 1.3, 1.3774, 1.378
(d) 0.0044, 0.044, 0.4, 0.44
(e) 0.1, 0.109, 0.110, 0.112

6.

		hundred	ten	tenth	hundredth
(a)	876.432	900	880	876.4	876.43
(b)	1634.075	1600	1630	1634.1	1634.08
(c)	189.6387	200	190	189.6	189.64
(d)	108.999	100	110	109.0	109.00
(e)	545.454	500	550	545.5	545.45

7. (a) 8 000 030 007
(b) 0.9
(c) 63 400 000
(d) 70.08
(e) 7.53
(f) 203.7
(g) 9007.623
(h) 0.004

Page 24

1. $$\begin{array}{r} 3.478 \\ 12.961 \\ \underline{100.003} \\ 116.442 \end{array}$$

2. $$\begin{array}{r} 16.65 \\ 123.08 \\ \underline{5.97} \\ 145.70 \end{array}$$

Page 25

1. $$\begin{array}{r} 18.63 \\ \underline{-9.48} \\ 9.15 \end{array}$$

2. $$\begin{array}{r} 123.7 \\ \underline{16.9} \\ 106.8 \end{array}$$

3. $$\begin{array}{r} 7.001 \\ \underline{5.635} \\ 1.366 \end{array}$$

Page 28

...the decimal point? **5**

Page 29

...in the product? **one**

Page 33

$$\begin{array}{r} 106 \\ 32\overline{)3392} \\ \underline{32} \\ 192 \\ \underline{192} \\ 0 \end{array}$$

$$\begin{array}{r} 350 \\ 121\overline{)42350} \\ \underline{363} \\ 605 \\ \underline{605} \\ 00 \\ \underline{00} \\ 0 \end{array}$$

$$\begin{array}{r} 1827 \\ 20\overline{)36547} \\ \underline{20} \\ 165 \\ \underline{160} \\ 54 \\ \underline{40} \\ 147 \\ \underline{140} \\ 7 \end{array}$$

Page 36

$$\begin{array}{r} 0.052 \\ 136\,\overline{)7.072} \\ \underline{680} \\ 272 \\ \underline{272} \\ 0 \end{array} \qquad \begin{array}{r} 12.09 \\ 21\,\overline{)253.89} \\ \underline{21} \\ 43 \\ \underline{42} \\ 189 \\ \underline{189} \\ 0 \end{array}$$

$$\begin{array}{r} 0.05 \\ 230\,\overline{)11.50} \\ \underline{11.50} \\ 0 \end{array} \qquad \begin{array}{r} 0.0031 \\ 234\,\overline{)0.7254} \\ \underline{702} \\ 234 \\ \underline{234} \\ 0 \end{array}$$

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$0.2415 \div 0.035 = \mathbf{6.9}$

Page 38

1. $$\begin{array}{r} 400. \\ 0.13\,\overline{)5200} \\ \underline{52} \\ 000 \end{array}$$

2. $$\begin{array}{r} 0.267 \\ 1.5\,\overline{)0.4005} \\ \underline{30} \\ 100 \\ \underline{90} \\ 105 \\ \underline{105} \\ 0 \end{array}$$

3. $$\begin{array}{r} 6.3 \\ 0.315\,\overline{)1.9845} \\ \underline{1890} \\ 945 \\ \underline{945} \\ 0 \end{array}$$

4. $$\begin{array}{r} 0.056 \\ 0.93\,\overline{)0.05208} \\ \underline{465} \\ 558 \\ \underline{558} \\ 0 \end{array}$$

Page 40

1. (a)

$$\begin{array}{r} 16.023 \\ 4.2_{\wedge}\overline{)67.3_{\wedge}000} \\ \underline{42} \\ 253 \\ \underline{252} \\ 100 \\ \underline{84} \\ 160 \\ \underline{126} \\ 34 \end{array}$$

Rounded quotient is 16.024

(b)

$$\begin{array}{r} 0.028 \\ 8.4_{\wedge}\overline{)0.2_{\wedge}38} \\ \underline{168} \\ 700 \\ \underline{672} \\ 28 \end{array}$$

Rounded quotient is 0.028

2. (a)

$$\begin{array}{r} 60.27 \\ 0.36_{\wedge}\overline{)21.70_{\wedge}00} \\ \underline{216} \\ 100 \\ \underline{72} \\ 280 \\ \underline{252} \\ 28 \end{array}$$

Rounded quotient is 60.3

(b)

$$\begin{array}{r} 6.38 \\ 5.1_{\wedge}\overline{)32.5_{\wedge}8} \\ \underline{306} \\ 198 \\ \underline{153} \\ 450 \\ \underline{408} \\ 42 \end{array}$$

Rounded quotient is 6.4

Page 42

...to move the decimal point? **4 places**

...place holders in the quotient? **one**

Page 43

Exercise – Operating with Decimal Numbers

1. (a) sum, right
   (b) divisor, caret, right, dividend, dividend
   (c) 4, right
   (d) 3, left
   (e) quotient
   (f) product

2. (a) $\frac{7}{10}$ (b) $\frac{29}{100}$

(c) $\frac{1}{1000}$ (d) $\frac{17}{10\,000}$

3. (a) $12\frac{1}{10}$ (b) $132\frac{67}{100}$

(c) $6\frac{99}{100}$ (d) $100\frac{3}{100}$

4. (a)
$$\begin{array}{r} 62.49 \\ 894.71 \\ 123.62 \\ 673.00 \\ 86.43 \\ \hline 1840.25 \end{array}$$
(b)
$$\begin{array}{r} 94.3 \\ 86.7 \\ 125.2 \\ 13.6 \\ 19.9 \\ \hline 339.7 \end{array}$$

(c)
$$\begin{array}{r} 132.456 \\ 2.015 \\ 37.916 \\ 0.070 \\ \hline 172.457 \end{array}$$
(d)
$$\begin{array}{r} 3.0156 \\ 0.1037 \\ 0.0069 \\ 0.0004 \\ \hline 3.1266 \end{array}$$

5. (a)
$$\begin{array}{r} 5.709 \\ 0.250 \\ \hline 5.459 \end{array}$$
(b)
$$\begin{array}{r} 27.9785 \\ 17.0436 \\ \hline 10.9349 \end{array}$$

(c)
$$\begin{array}{r} 30.63 \\ 29.18 \\ \hline 1.45 \end{array}$$
(d)
$$\begin{array}{r} 7.28364 \\ 1.30567 \\ \hline 5.97797 \end{array}$$

6. (a)
$$\begin{array}{r} 32 \\ \times 0.04 \\ \hline 1.28 \end{array}$$
(b)
$$\begin{array}{r} 17.23 \\ \times 10.9 \\ \hline 15507 \\ 0000 \\ 1723 \\ \hline 187.807 \end{array}$$

(c)

$$\begin{array}{r} 1.057 \\ 0.035 \\ \hline 5285 \\ 3171 \\ \hline 0.036995 \end{array}$$

7. (a)

$$\begin{array}{r} 0.045 \\ 13\overline{)0.585} \\ 52 \\ \hline 65 \\ 65 \\ \hline 0 \end{array}$$

(b)

$$\begin{array}{r} 4.3 \\ 36\overline{)154.8} \\ 144 \\ \hline 10\,8 \\ 10\,8 \\ \hline 0 \end{array}$$

(c)

$$\begin{array}{r} 1.06 \\ 42.1_\wedge\overline{)44.6_\wedge26} \\ 421 \\ \hline 2526 \\ 2526 \\ \hline 0 \end{array}$$

(d)

$$\begin{array}{r} 540. \\ 0.362_\wedge\overline{)195.480_\wedge} \\ 1810 \\ \hline 1448 \\ 1448 \\ \hline 0 \end{array}$$

(e)

$$\begin{array}{r} 0.15 \\ 0.062_\wedge\overline{)0.009_\wedge3} \\ 62 \\ \hline 310 \\ 310 \\ \hline 0 \end{array}$$

(f)

$$\begin{array}{r} 0.0153 \\ 0.12_\wedge\overline{)0.00_\wedge1836} \\ 12 \\ \hline 63 \\ 60 \\ \hline 36 \\ 36 \\ \hline 0 \end{array}$$

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8. (a)

$$\begin{array}{r} 95.8 \\ 0.073_\wedge\overline{)7000_\wedge} \\ 657 \\ \hline 430 \\ 365 \\ \hline 650 \\ 584 \\ \hline 66 \end{array}$$

Rounded quotient is 95.9

(b)

$$\begin{array}{r} 3.2 \\ 21.3_\wedge\overline{)69.5_\wedge} \\ 639 \\ \hline 560 \\ 426 \\ \hline 134 \end{array}$$

Rounded quotient is 3.3

9. (a)

$$\begin{array}{r} 5.408 \\ 2.4\overline{)12.9800} \\ \underline{120} \\ 98 \\ \underline{96} \\ 200 \\ \underline{192} \\ 8 \end{array}$$

Rounded quotient is 5.408

(b)

$$\begin{array}{r} 0.013 \\ 6.8\overline{)0.0900} \\ \underline{68} \\ 220 \\ \underline{204} \\ 16 \end{array}$$

Rounded quotient is 0.013

10. (a) 2.7 (b) 140
(c) 0.04213 (d) 0.0013
(e) 710 (f) 0.00437
(g) 0.004 (h) 340
(i) 28 900 (j) 30
(k) 0.00171 (l) 0.24

END OF LESSON 5

LESSON 6

Page 1

2. $\dfrac{7}{20} = 0.35$

$$\begin{array}{r} 0.35 \\ 20\,\overline{)7.00} \\ \underline{60} \\ 100 \\ \underline{100} \\ 0 \end{array}$$

3. $\dfrac{-5}{16} = -0.3125$

$$\begin{array}{r} 0.3125 \\ 16\,\overline{)5.0000} \\ \underline{48} \\ 20 \\ \underline{16} \\ 40 \\ \underline{32} \\ 80 \\ \underline{80} \\ 0 \end{array}$$

4. $\dfrac{43}{4} = 10.75$

$$\begin{array}{r} 10.75 \\ 4\,\overline{)43.00} \\ \underline{4} \\ 30 \\ \underline{28} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

Page 3

2. $\dfrac{5}{18} = 0.2\overline{7}$

$$\begin{array}{r} 0.2777 \\ 18\,\overline{)5.0000} \\ \underline{36} \\ 140 \\ \underline{126} \\ 140 \end{array}$$

3. $\dfrac{-31}{15} = -2.0\overline{6}$

$$\begin{array}{r} 2.0666 \\ 15\,\overline{)31.0000} \\ \underline{30} \\ 100 \\ \underline{90} \\ 100 \\ \underline{90} \\ 100 \end{array}$$

4. $\frac{5}{11} = .\overline{45}$

$$\begin{array}{r} 0.4545 \\ 11\overline{)5.000} \\ \underline{44} \\ 60 \\ \underline{55} \\ 50 \\ \underline{44} \\ 60 \\ \underline{55} \\ 5 \end{array}$$

Page 4

1. $\frac{-309}{10\ 000}$
2. $\frac{9}{10}$
3. $\frac{73}{100}$
4. $\frac{-17}{1000}$
5. $\frac{7}{100}$
6. $\frac{-23}{10\ 000}$

	Decimal	Mixed Number	Rational Number
2.	10.7	$10\frac{7}{10}$	$\frac{107}{10}$
3.	−1.03	$-1\frac{3}{100}$	$\frac{-103}{100}$
4.	1.901	$1\frac{901}{1000}$	$\frac{1901}{1000}$
5.	66.67	$66\frac{67}{100}$	$\frac{6667}{100}$

Page 5

$$x = \frac{12}{9} \text{ or } \frac{\mathbf{4}}{\mathbf{3}}$$

$\therefore$ the rational number $\frac{\mathbf{4}}{\mathbf{3}}$.

$$x = \frac{732}{999} \text{ or } \frac{\mathbf{244}}{\mathbf{333}}$$

$\therefore$ the rational number $\frac{\mathbf{244}}{333}$.

Page 6

1. $$\begin{aligned} 100x &= 423.2323\ldots \\ x &= 4.2323\ldots \\ \hline 99x &= 419 \end{aligned}$$

$$x = \frac{419}{99}$$

$$-4.\overline{23} = -\frac{419}{99}$$

2. $$\begin{aligned} 10x &= 1.1222\ldots \\ x &= .1122\ldots \\ \hline 9x &= 1.01 \end{aligned}$$

$$x = \frac{1.01}{9} = \frac{101}{900}$$

$$0.11\overline{2} = \frac{101}{900}$$

Page 7

2. commutative property of addition
3. additive identity is 0
4. multiplicative inverses
5. associative property of multiplication
6. additive inverses

Exercise – Relationship Between Decimal Numbers and Rational Numbers

1. (a) periodic
   (b) denominator, numerator
   (c) terminating
   (d) rational
   (e) terminating repeating
   (f) $\frac{37}{100}$
   (g) $2.\overline{3}$

2. (b) 1.625, terminating
   (c) $0.08\overline{3}$, non-terminating repeating
   (d) $0.7\overline{3}$, non-terminating repeating
   (e) 0.225, terminating

(b)

$$\begin{array}{r} 1.625 \\ 8\,\overline{)13.000} \\ \underline{8} \\ 50 \\ \underline{48} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

(c)

$$\begin{array}{r} 0.0833 \\ 12\,\overline{)1.0000} \\ \underline{96} \\ 40 \\ \underline{36} \\ 40 \\ \underline{36} \\ 4 \end{array}$$

(d)

$$\begin{array}{r} 0.733 \\ 15\,\overline{)11.000} \\ \underline{105} \\ 50 \\ \underline{45} \\ 50 \\ \underline{45} \\ 5 \end{array}$$

(e)

$$\begin{array}{r} 0.225 \\ 40\,\overline{)9.000} \\ \underline{80} \\ 100 \\ \underline{80} \\ 200 \\ \underline{200} \\ 0 \end{array}$$

3. (a)

$$\begin{aligned} 100x &= 45.4545\ldots \\ x &= .4545\ldots \\ \hline 99x &= 45 \\ x &= \frac{45}{99} = \frac{5}{11} \end{aligned}$$

(b) $\dfrac{164}{1000} = \dfrac{41}{250}$

(c)

$$\begin{aligned} 1000x &= 123.123123\ldots \\ x &= 0.123123\ldots \\ 999x &= 123 \\ x &= \frac{123}{999} = \frac{41}{333} \end{aligned}$$

(d) $\dfrac{25}{10\ 000} = \dfrac{1}{400}$

Page 10

3.	periodic	rational
4.	non-periodic	irrational
5.	periodic	rational
6.	periodic	rational
7.	periodic	rational
8.	periodic	rational
9.	non-periodic	irrational
10.	periodic	rational

Page 11

1. principal , 16
   $\underline{4}$ since $4 \times \underline{4} = 16$

2. negative , 81
   $\underline{-9}$ since $-9 \times \underline{-9} = 81$

3. $\sqrt{27}$, $\sqrt{27}$, $-\sqrt{27}$

4. -5, 5, -5

i.e.: $\sqrt{121} = \mathbf{11}$

$-\sqrt{121} = \mathbf{-11}$

$\sqrt{2500} = \mathbf{50}$

$-\sqrt{0.36} = \mathbf{-0.6}$

Name five other real numbers that are perfect squares.

8100, 4, 9, 25, 16, (many other perfect square)

Page 12

		Check
2.	$\sqrt{144} = 12$	$12 \times 12 = 144$
3.	$\sqrt{0.49} = 0.7$	$0.7 \times 0.7 = 0.49$
4.	$\sqrt{0.0016} = 0.04$	$0.04 \times 0.04 = 0.0016$
5.	$-\sqrt{\frac{9}{16}} = -\frac{3}{4}$	$-\frac{3}{4} \times -\frac{3}{4} = \frac{9}{16}$
6.	$\sqrt{10\,000} = 100$	$100 \times 100 = 10\,000$

Name five other square roots that are irrational numbers:

$\sqrt{65}$, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$, (or any other similar answer)

Page 14

$(BD)^2 = (\mathbf{1})^2 + (1)^2$

Page 15

...the irrational number $-\sqrt{2}$
What two integers does $\sqrt{2}$ be between? **1 and 2**
Which of these integers is it closer to? **1**
What two integers does $-\sqrt{2}$ lie between? **−1 and −2**
Which of these integers is it closer to? **−1**
$\therefore$ on the number line **3 and 4**...is it closer to? **3**

Page 16

2. 2.74
3. 2.733

Page 17

1. (b) false
   (c) true
   (d) true
   (e) false
   (f) true
   (g) true

2. (b) true
   (c) false
   (d) false
   (e) true
   (f) false
   (g) true

Page 18

Square Root	Decimal Value	Exact or Approximate
$\sqrt{49}$	7	exact
$\sqrt{100}$	10	exact
$-\sqrt{68}$	−8.246	approximate
$\sqrt{24}$	4.899	approximate
$\sqrt{1}$	1	exact
$-\sqrt{72}$	−8.485	approximate

Page 19

1. $\doteq 13 + 2.646$
   $\doteq 15.646$

2. $\doteq 3 \times 4.243$
   $\doteq 12.729$

3. $\doteq (3 \times 2.449) + (5 \times 2.828)$
   $\doteq 7.347 + 14.140$
   $\doteq 21.487$

4. $\doteq \dfrac{(9.695) - (9.434)}{3}$
   $\doteq \dfrac{0.261}{3} \doteq 0.087$

Page 23

Is $\sqrt{2}$ included in the set that is graphed above? **Yes**

Is $-\sqrt{2}$? **Yes** Is $-\sqrt{9}$? **Yes** Is $-\sqrt{10}$? **No**

Is 4.65? **Yes** Is $\sqrt{87}$? **Yes** Is 0? **Yes**

Is -0.3? **Yes** Is $\frac{18}{5}$? **Yes** Is $-\pi$? **No**

Page 24

Note: The answers for page 24 must be set up in the exact order shown and they must be in four columns.

Is $\frac{4}{3}$? **No**	Is $-\sqrt{6}$? **Yes**	Is $-\pi$? **Yes**	Is π? **No**
Is $\sqrt{2}$? **No**	Is $-\sqrt{25}$? **No**	Is $\sqrt{1}$? **Yes**	
Is $\frac{-39}{8}$? **Yes**	Is 0.3? **Yes**		

Page 25

	Addition	Multiplication
Closure Properties	a + b is a real number	**ab** is a **real** number
Commutative Properties	a + b = b + a	ab = **ba**
Associative Properties	a + (b + c) = **(a + b)** + c	**a(bc)** = **(ab)c**
Distributive Property	a(b + c) = **ab** + **ac**	
Identity Elements	Additive identity is **0.** a + **0** = a	Multiplicative identity is **1.** a × **1** = a
Inverse Elements	Additive inverse of a is **−a**	Multiplicative inverse of a is $\mathbf{\frac{1}{a}}$
	a + **(−a)** = 0	a × $\mathbf{\frac{1}{a}}$ = 1

Page 26

1. Does 0 belong to set N? **No** Does 5? **Yes** Does -2? **No**
   Does 1000? **Yes** Does $3\frac{1}{2}$? **No** Does $\sqrt{3}$? **No**

2. 0

3. Does 0 belong to set I? **Yes** Does 5? **Yes** Does -5? **Yes**
   Does $2\frac{1}{2}$? **No** Does $\sqrt{2}$? **No** Does -4.2? **No**
   Is 2 a natural number? **Yes** a whole number? **Yes** an integer? **Yes**
   Is -2 a natural number? **No** a whole number? **No** an integer? **Yes**
   Is 0 a rational number? **No** a whole number? **Yes** an integer? **Yes**

4. Does $\frac{1}{4}$ belong to set Q? **Yes** Does 3.98? **Yes** Does $\sqrt{2}$? **No**
   Does π? **No** Does $\frac{39}{7}$? **Yes** Does $106.\overline{8}$? **Yes**
   Does 1.43678...? **No** Does $\frac{-18}{3}$? **Yes** Does $2\frac{3}{4}$? **Yes**
   Is 7 a natural number? **Yes** a whole number? **Yes** an integer? **Yes**
   a rational number? **Yes**
   Is -5 a natural number? **No** an integer **Yes** a rational number? **Yes**
   Is $\frac{1}{3}$ a whole number? **No** an integer? **No** a rational number? **Yes**
   Is $0.\overline{6}$ a whole number? **No** an integer? **No** a rational number? **Yes**

5. Is $\sqrt{12}$ a natural number? **No** an integer? **No** a rational number? **No**
   an irrational number? **Yes**
   Is $\sqrt{9}$ a natural number? **Yes** a rational number? **Yes** a irrational number? **No**
   Is 2.03003 a natural number? **No** an integer? **No** a rational number? **No**
   an irrational number? **Yes**

6. Is 0 a real number? **Yes** Is -1? **Yes** Is 236? **Yes**
   Is $\frac{1}{2}$? **Yes** Is $-\frac{1}{2}$? **Yes** Is $3\frac{1}{2}$? **Yes**
   Is π? **Yes** Is $\sqrt{3}$? **Yes** Is $-\sqrt{3}$? **Yes**
   Is $-\sqrt{3}$? **No** Is 1.3? **Yes** Is 1.3674...? **Yes**
   Is $-\sqrt{-1}$? **No**

Page 30

Exercise – The Real Number System

1. (a) density
   (b) completeness
   (c) rational, irrational
   (d) real
   (e) rational, real
   (f) irrational
   (g) non-periodic (or non-terminating)
   (h) rational, irrational
   (i) right
   (j) irrational
   (k) real, rational
   (l) addition, commutative
   (m) $-\sqrt{10}$
   (n) 1
   (o) positive, absolute
   (p) positive (or principal), negative
   (q) perfect, 11
   (r) -2, -3

2. (a) $<$ (b) $>$
   (c) $=$ (d) $>$
   (e) $=$ (f) $>$
   (g) $=$ (h) $<$
   (i) $<$ (j) $<$
   (k) $>$ (l) $>$
   (m) $<$ (n) $>$
   (o) $<$ (p) $<$
   (q) $>$ (r) $<$

3. (a) 1 (b) $\sqrt{2}$
   (c) 5 (d) 1
   (e) 0 (f) 0
   (g) $\sqrt{8}$ (h) $\sqrt{3}$
   (i) $\sqrt{2}$ (j) $\frac{2}{3}$

4.

	N	W	I	Q	R
(a) $\frac{-2}{7}$				✓	✓
(b) 0		✓	✓	✓	✓
(c) 16	✓	✓	✓	✓	✓
(d) $5.\overline{3}$				✓	✓
(e) $\sqrt{11}$					✓
(f) $-\pi$					✓
(g) $-\sqrt{4}$			✓	✓	✓
(h) $\sqrt{-4}$					
(i) 8.6143...					✓
(j) -1			✓	✓	✓
(k) 12.35				✓	✓
(l) $-8\frac{1}{6}$				✓	✓

5.

	N	W	I	Q	R
(a)	✓	✓	✓	✓	✓
(b)	✓	✓	✓	✓	✓
(c)	✓	✓	✓	✓	✓
(d)	✓	✓	✓	✓	✓
(e)	✓	✓	✓	✓	✓
(f)	✓	✓	✓	✓	✓
(g)	✓	✓	✓	✓	✓
(h)		✓	✓	✓	✓
(i)	✓	✓	✓	✓	✓
(j)			✓	✓	✓
(k)				✓	✓

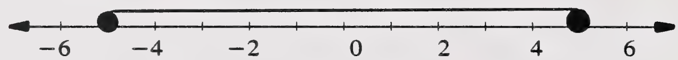
6. (a)	$\sqrt{8}$	✓	(b)	$\sqrt{-23}$	
(c)	$\frac{-3}{7}$		(d)	$\sqrt{\frac{1}{4}}$	
(e)	$-\pi$	✓	(f)	$6.0\overline{8}$	
(g)	$-4\frac{1}{8}$		(h)	1.632...	✓
(i)	$\sqrt{\frac{1}{7}}$	✓	(j)	$\sqrt{\frac{-1}{7}}$	
(k)	$-\sqrt{81}$		(l)	$-\sqrt{2.3}$	✓

7. (a)	-6	✓	(b)	$2\frac{1}{2}$	
(c)	$-\sqrt{64}$	✓	(d)	$\sqrt{-81}$	
(e)	$\frac{0}{4}$	✓	(f)	$\frac{-4}{8}$	
(g)	$\frac{-9}{3}$	✓	(h)	$\sqrt{\frac{9}{16}}$	
(i)	0.3				

8. (a)	-4	✓	(b)	$1\frac{1}{6}$	✓
(c)	0	✓	(d)	π	
(e)	3.51	✓	(f)	0.12112...	
(g)	$0.\overline{7}$	✓	(h)	$\sqrt{26}$	
(i)	$\sqrt{\frac{4}{81}}$	✓	(j)	$-\sqrt{0.09}$	✓
(k)	$\sqrt{3.15}$		(l)	133	✓

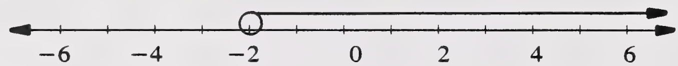
9. (a) Set-builder notation: $\{x \mid -5 \leq x \leq 5, x \in R\}$

Graph:



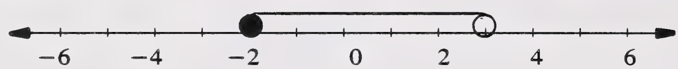
(b) Set-builder notation: $\{x \mid x > -2, x \in R\}$

Graph:



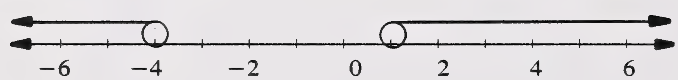
(c) Set-builder notation: $\{x \mid -2 \leq x < 3, x \in R\}$

Graph:



(d) Set-builder notation: $\{x \mid x < -4 \text{ or } x > 1, x \in R\}$

Graph:



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2. $\frac{3}{4}$

3. $\sqrt{3}$, 3 (or any other number)

Give examples of five other mixed radicals.

$\frac{1}{2}\sqrt{2}$, $2\sqrt{3}$, $3\sqrt{5}$, $5\sqrt{6}$, $3\sqrt{2}$ (or any other example)

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1. $\sqrt{66}$

2. $\sqrt{2 \times 15} = \sqrt{30}$

3. $\sqrt{5 \times 3} = \sqrt{15}$

4. $\sqrt{7 \times 10} = \sqrt{70}$

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2. $\sqrt{7 \times 15} = \sqrt{105}$

3. $-\sqrt{6 \times 13} = -\sqrt{78}$

4. $+\sqrt{17 \times 2} = \sqrt{34}$

1. 10

2. 7

3. 0.1

4. $\frac{1}{8}$

5. $\frac{2}{3}$

6. 62.4

7. 11

8. 9

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1. $-\sqrt{5 \times 6 \times 7} = -\sqrt{210}$

2. $\sqrt{0.75 \times 2 \times 6} = \sqrt{9} = 3$

3. $-\sqrt{0.1 \times 6 \times 7} = -\sqrt{4.2}$

4. $-\sqrt{2 \times 0.1 \times 3 \times 0.2 \times 5} = -\sqrt{0.6}$

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$(6 \times -3)\sqrt{5 \times 2} = -18\sqrt{10}$

$(-5 \times -8)\sqrt{2 \times 3} = 40\sqrt{6}$

$\left(\frac{2}{3} \times \frac{1}{2}\right)\sqrt{11 \times 7} = \frac{1}{3}\sqrt{77}$

$-2\sqrt{2} \times 5\sqrt{3} \times 6\sqrt{5} = (-2 \times 5 \times 6)\sqrt{2 \times 3 \times 5}$

$= -60\sqrt{30}$

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1. $\sqrt{\frac{77}{7}} = \sqrt{11}$
2. $\sqrt{\frac{182}{13}} = \sqrt{14}$
3. $-\sqrt{\frac{110}{22}} = -\sqrt{5}$

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1. $\frac{-9}{3} \times \frac{\sqrt{6}}{\sqrt{2}} = -3\sqrt{3}$
2. $\frac{12}{16} \times \sqrt{\frac{95}{5}} = \frac{3}{4}\sqrt{19}$
3. $\frac{-14}{-2} \times \sqrt{\frac{2}{2}} = 7 \times \sqrt{1} = 7$

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$(2 - 6 + 9 - 15)\sqrt{5} = -10\sqrt{5}$

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2. $\sqrt{100 \times 3} = 100 \times \sqrt{3} = 10\sqrt{3}$

3. $\sqrt{81 \times 5} = \sqrt{81} \times \sqrt{5} = 9\sqrt{5}$

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4, 9, **16**, 25, 36, **49**, **64**, 81, 100, **121**, 144

$2\sqrt{8} + 3\sqrt{50} + \sqrt{128} = 2\sqrt{4 \times 2} + 3\sqrt{25 \times 2} + \sqrt{64 \times 2}$

$= 2\sqrt{4}\sqrt{2} + 3\sqrt{25}\sqrt{2} + \sqrt{64}\sqrt{2}$
$= 2 \times 2\sqrt{2} + 3 \times 5\sqrt{2} + 8\sqrt{2}$
$= 4 \times \sqrt{2} + 15\sqrt{2} + 8\sqrt{2}$
$= (4 + 15 + 8)\sqrt{2}$
$= 27\sqrt{2}$

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Exercise – Operating With Radicals

1. (a) radicands
   (b) mixed
   (c) 25, 5
   (d) radicand
   (e) fraction
   (f) identicals
   (g) denominator, $\sqrt{7}$

2. (a) $\sqrt{4} \times \sqrt{3} = 2\sqrt{3}$
   (b) $\sqrt{36 \times 2} = \sqrt{36} \times \sqrt{2} = 6 \times \sqrt{2} = 6\sqrt{2}$
   (c) $\sqrt{16 \times 5} = \sqrt{16} \times \sqrt{5} = 4 \times \sqrt{5} = 4\sqrt{5}$
   (d) $\sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2} = 5 \times \sqrt{2} = 5\sqrt{2}$
   (e) 11
   (f) $\sqrt{36 \times 2} = \sqrt{36} \times \sqrt{2} = 6 \times \sqrt{2} = 6\sqrt{2}$
   (g) $\sqrt{100 \times 10} = \sqrt{100} \times \sqrt{10} = 10 \times \sqrt{10} = 10\sqrt{10}$
   (h) $2\sqrt{100 \times 5} = 2\sqrt{100} \times \sqrt{5} = 2 \times 10 \times \sqrt{5} = 20\sqrt{5}$

3. (a) 7

   (b) $\frac{-2}{3}$

   (c) $5 \times -3 \times 6 = -90$

   (d) $18\sqrt{55}$

   (e) $5 \times 11 \times \sqrt{3 \times 15} = 55\sqrt{45} = 55\sqrt{9 \times 5} = 55 \times 3\sqrt{5} = 165\sqrt{5}$

   (f) $\sqrt{\frac{82}{2}} = \sqrt{41}$

   (g) $\frac{-5}{3}$

   (h) $2\sqrt{\frac{15}{5}} = 2\sqrt{3}$

   (i) $2 \times 3 \times 7 = 42$

(j) $\frac{6}{-2} \times \sqrt{\frac{55}{5}} = -3\sqrt{11}$

(k) $-6\sqrt{2}$

(l) $(8 - 2 + 1)\sqrt{5} = 7\sqrt{5}$

(m) $2\sqrt{3} + 3\sqrt{6} = (2 + 3)\sqrt{6} = 5\sqrt{6}$

(n) $(3 \times 5)\sqrt{2 \times 3} + (8 \times -2)\sqrt{6} = 15\sqrt{6} - 16\sqrt{6} = (15 - 16)\sqrt{6} = -\sqrt{6}$

(o) $\sqrt{4 \times 3} + 3\sqrt{9 \times 3} - 2\sqrt{25 \times 3}$
$= 2\sqrt{3} + 9\sqrt{3} - 10\sqrt{3} = (2 + 9 - 10)\sqrt{3} = 1\sqrt{3} = \sqrt{3}$

(p) $3\sqrt{36 \times 2} + 5\sqrt{16 \times 2} - 6\sqrt{25 \times 2}$
$= 3 \times 6\sqrt{2} + 5 \times 4\sqrt{2} - 6 \times 5\sqrt{2}$
$= 18\sqrt{2} + 20\sqrt{2} - 30\sqrt{2}$
$= (18 + 20 - 30)\sqrt{2} = 8\sqrt{2}$

4. (a) 12.124

(b) $\sqrt{325} = \sqrt{25 \times 13}$
$= \sqrt{25} \times \sqrt{13}$
$= 5\sqrt{13}$
$\doteq 5 \times 3.606$
$\doteq 18.030$

(c) $\sqrt{448} = \sqrt{64 \times 7}$
$= \sqrt{64} \times \sqrt{7}$
$= 8\sqrt{7}$
$\doteq 8 \times 2.646$
$\doteq 21.168$

(d) $\sqrt{242} = \sqrt{121 \times 2}$
$= \sqrt{121} \times \sqrt{2}$
$= 11\sqrt{2}$
$\doteq 11 \times 1.414$
$\doteq 15.554$

(e) $\sqrt{1100} = \sqrt{100 \times 11}$
$= 10\sqrt{10}$
$\doteq 10 \times 3.317$
$\doteq 33.17$

(f) $\sqrt{212} = \sqrt{4 \times 53}$
$= \sqrt{4} \times \sqrt{53}$
$= 2\sqrt{53}$
$\doteq 2 \times 7.280$
$\doteq 14.560$

END OF LESSON 6

LESSON 7

Page 1

With respect to the power 3^4, what is the base of the power? **3**

What is the exponent? **4** This power tells us that the number **3** is to be used **4** times as a factor.

i.e.: $3^4 = 3 \times 3 \times 3 \times 3 = \mathbf{81}$

Write $(7 \times 7 \times 7 \times 7 \times 7 \times 7)$ in exponential form. $\mathbf{7^6}$

i.e.: $(\sqrt{2})^2 = \sqrt{2} \times \sqrt{2} = \mathbf{2}$

i.e.: $(-3)^3 = (-3)(-3)(-3) = \mathbf{-27}$

i.e.: $(\frac{-2}{5})^4 = \frac{-2}{5} \times \frac{-2}{5} \times \frac{-2}{5} \times \frac{-2}{5} = \mathbf{\frac{16}{625}}$

Page 2

How would you write -2 to the fifth? $\mathbf{(-2)^5}$

-7 squared? $\mathbf{(-7)^2}$

What does $(-3)^4$ equal? **81**

What does -3^4 equal? **−81**

2. $5^4 = 5 \times 5 \times 5 \times 5 = 625$

2. $(-3)(-3)(-3)(-3) = 81$

3. $(-1)^6 = (-1)(-1)(-1)(-1)(-1)(-1) = 1$

2. $(-3)^5 = (-3)(-3)(-3)(-3)(-3) = -243$

3. $(-1)^7 = (-1)(-1)(-1)(-1)(-1)(-1)(-1) = -1$

$(-16)^8$, $(-2)^2$, $(-3)^4$, $(-6)^2$ (any other similar example)

$(-16)^7$, $(-2)^3$, $(-6)^1$, $(-3)^5$ (any other similar example)

Page 3

How would you write $\frac{1}{2}$ squared? $\left(\frac{1}{2}\right)^2$, $\frac{-3}{8}$ to the fourth? $\left(\frac{-3}{8}\right)^4$

What does $\left(\frac{2}{3}\right)^5$ equal? $\mathbf{\frac{32}{243}}$

What does $\frac{2^5}{3}$ equal? $\mathbf{\frac{32}{3}}$

$(-2)^3 + 3^2 = (-2 \times -2 \times -2) + (3 \times 3)$
$= \mathbf{-8 + 9}$
$= \mathbf{1}$

Page 4

Which of the powers above have bases that are natural numbers?

$\mathbf{3^2, 9^8}$

Which have bases that are integers? 3^2, $\mathbf{(-2)^5, 9^8}$

Which have bases that are rational numbers? 3^2, $\mathbf{(-2)^5, \left(\frac{1}{2}\right)^4, (2.5)^4, 9^8}$

Which have bases that are irrational numbers? π^3, $(\sqrt{2})^7$

Exercise – Positive Integral Powers

1. (a) base, exponent
   (b) −9, factor
   (c) power (or exponential)
   (d) negative, fractional
   (e) even
   (f) real, positive

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	Power	Base	Exponent	Evaluation
2. (b)	9^3	9	3	$9 \times 9 \times 9 = 729$
(c)	$(-8)^3$	−8	3	$-8 \times -8 \times -8 = -512$
(d)	$(\frac{-5}{6})^2$	$\frac{-5}{6}$	2	$\frac{-5}{6} \times \frac{-5}{6} = \frac{25}{36}$
(e)	$(\frac{-4}{7})^3$	$\frac{-4}{7}$	3	$\frac{-4}{7} \times \frac{-4}{7} \times \frac{-4}{7} = \frac{-64}{343}$
(f)	$(\sqrt{2})^3$	$\sqrt{2}$	3	$\sqrt{2} \times \sqrt{2} \times \sqrt{2} = 2\sqrt{2}$
(g)	$(-\sqrt{3})^4$	$-\sqrt{3}$	4	$-\sqrt{3} \times -\sqrt{3} \times -\sqrt{3} \times -\sqrt{3} = 9$
(h)	$(-1)^{18}$	−1	18	$(-1)^{18} = 1$
(i)	$(-1)^{33}$	−1	33	$(-1)^{33} = -1$

3. (a) negative (b) positive
(c) negative (d) negative
(e) positive (f) negative
(g) positive (h) negative
(i) negative (j) negative
(k) positive (l) negative

4. (b) $(-6)^4$

(c) $8^4\ 5^3\ (-1)^2$

(d) $(-3)^2\ (3)^3\ (2)^2$

(e) $\left(\frac{-1}{2}\right)^4\ \left(\frac{1}{3}\right)^3$

(f) $\left(\frac{-4}{3}\right)^5\ \left(\frac{3}{5}\right)^2$

5. (a) $= 49 - (-8)$
$= 49 + 8$
$= 57$

(b) $= 9 - 16$
$= -7$

(c) $= -\frac{1}{8} - \frac{9}{64}$

$= \frac{-8 - 9}{64}$

$= \frac{-17}{64}$

(d) $= -5\ (81 - 64)$

$= -\ 5(17)$

$= -85$

(e) $\frac{-5}{2} \div \frac{1}{4}$

$= \frac{-5}{1} \times \frac{4}{1}$

$= -10$

(f) $= \frac{-1 + 16}{3}$

$= \frac{+15}{3}$

$= 5$

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2. $6^{5+2} = 6^7$

3. $\left(\frac{-2}{3}\right)^{2+3} = \left(\frac{-2}{3}\right)^5$

4. $(-8)^{1+7} = (-8)^8$

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1. $3^3 \times 3^7$ ___✓___ 2. $6^7 \times 7^6$ ______

3. $(-2)^5\ 5$ ______ 4. $\left(\frac{1}{2}\right)^2 \left(\frac{3}{2}\right)^3$ ______

5. $\left(\frac{-2}{3}\right) \left(\frac{-2}{3}\right)$ ______ 6. $(-3)^7 \left(\frac{1}{3}\right)^5$ ______

$(-3)^{6+4+1+7} = (-3)^{18}$ ______

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2. $(-4)^{10}(-7)^{10}$

3. $(-8)^4 \times 7^4$

4. $(-5)^3 \times \left(\frac{2}{3}\right)^3$

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$(-3)^3\ 2^3 \left(\frac{1}{2}\right)^3 \left(\frac{5}{7}\right)^3 (-4)^3$

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2. $(6)^{2 \times 15} = 6^{30}$

3. $\left(\frac{1}{2}\right)^{4 \times 5} = \left(\frac{1}{2}\right)^{20}$

4. $7^{3 \times 2} = 7^6$

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$2^{3 \times 2 \times 8} = 2^{48}$

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2. $\pi^{5-3} = \pi^2$

3. $(\sqrt{2})^{9-2} = (\sqrt{2})^7$

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2. $\frac{1}{(-6)^{9-5}} = \frac{1}{(-6)^4}$

3. $\frac{1}{(\sqrt{3})^4} = \frac{1}{(\sqrt{3})^1} = \frac{1}{\sqrt{3}}$

2. $17^{3-3} = 17^0 = 1$

3. $(\sqrt{6})^{2-2} = (\sqrt{6})^0 = 1$

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1.	1	2.	1
2.	1	3.	1
4.	1	4.	1
5.	1	6.	1

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Exercise – Power Properties

1. (a) base, add
   (b) 1
   (c) power of a power
   (d) multiply
   (e) larger
   (f) bases
   (g) power of a product

2.	D	1.	A	6.
	A	2.	F	7.
	E	3.	E	8.
	B	4.	D	9.
	C	5.	F	10.

3. (b) $(7.23)^{4+3} = (7.23)^7$

   (c) $5^{19} \times 1 = 5^{19}$

   (d) $(-2)^{6+3+1} = (-2)^{10}$

   (e) $\dfrac{7^{18}}{7^5} = 7^{18-5} = 7^{13}$

   (f) $(\sqrt{3})^{4-14} = (\sqrt{3})^{-10} = \dfrac{1}{(\sqrt{3})^{10}}$

   (g) $5^{3+5-2} = 5^6$

   (h) $7^{4+0+4-2-1} = 7^5$

   (i) $3^{12+3-1} = 3^{14}$

   (j) $\dfrac{(-6)^{1+7}}{(-6)^{4+5}} = \dfrac{(-6)^8}{(-6)^9} = \dfrac{1}{(-6)^{9-8}} = \dfrac{1}{(-6)^1} = \dfrac{1}{-6}$

(k) $\dfrac{5^{24}}{5^{10}} = 5^{24-10} = 5^{14}$

(l) $2^6 \times 2^{12} = 2^{6+12} = 2^{18}$

(m) $\dfrac{2^7}{2^5 \times 2^8} = \dfrac{2^7}{2^{5+8}} = \dfrac{2^7}{2^{13}} = \dfrac{1}{2^{13-7}} = \dfrac{1}{2^6}$

4. (a) $1 - 16 = -15$

(c) $\dfrac{1}{(-5)^{15-12}} = \dfrac{1}{(-5)^3} = \dfrac{1}{-125}$

(d) $2^0 - 2^2 + 2^4 = 1 - 4 + 16 = 13$

(e) $\dfrac{2^{20}}{2^{15}} = 2^{20-15} = 2^5 = 32$

(f) $-\dfrac{1}{27} + \dfrac{4}{9} = -\dfrac{1}{27} + \dfrac{12}{27} = \dfrac{-1+12}{27} = \dfrac{11}{27}$

5. (a) $(-5)^3 \times 3^3 \times 8^3$

(b) $\dfrac{\pi^2}{5^2}$

(c) $3^6 \times 5^{24}$

(d) $\dfrac{(-4)^{15}}{7^{10}}$

(e) $\dfrac{(-2)^8}{7^8}$

(f) $(-4)^{28} \times 3^8$

6. (b) $= \dfrac{5^{2+8} \times 3^4}{3^{9+3} \times 5^4} = \dfrac{5^{10}\, 3^4}{3^{12}\, 5^4}$

$= \dfrac{5^{10-4}}{3^{12-4}} = \dfrac{5^6}{3^8}$

(c) $\dfrac{6^{16} \times 5^{16}}{5^{12} \times 6^{10}}$

$= 6^{16-10}\, 5^{16-12}$

$= 6^6\, 5^4$

(d) $\frac{2^7}{3^7} \times \frac{3^4}{2^4}$

$= \frac{2^{7-4}}{3^{7-4}}$

$= \frac{2^3}{3^3}$

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Similarly, the power $\left(\frac{-3}{5}\right)^9$ tells us that the base $\frac{-3}{5}$ is to be used as a factor **nine** times.

What does $(2^3)^0$ equal? **1**

Give examples of three other negative integral powers?

$\mathbf{4^{-2}, 5^{-3}, 7^{-1}}$ (any other similar example.)

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2. $\frac{1}{5^2} = \frac{1}{25}$

3. $\frac{1}{(-2)^3} = \frac{1}{-8}$

4. 49

5. $1 \times 4 = 4$

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(a)	12	(b)	7
(c)	-2	(d)	0
(e)	-13	(f)	9

(b) $5^{-9+5} = 5^{-4}$

(c) $\left(\frac{1}{2}\right)^{-4+16} = \left(\frac{1}{2}\right)^{12}$

(d) $-(\sqrt{2})^{-12+-6} = (-\sqrt{2})^{-18}$

(e) $(-4)^{8-20} = (-4)^{-12}$

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(b) $(\sqrt{2})^{-7} \times (\sqrt{5})^{-7} = \frac{1}{(\sqrt{2})^7} \times \frac{1}{(\sqrt{5})^7}$

(c) $2^{-15} \times 3^{-15} \times (-7)^{-15} = \dfrac{1}{2^{15}} \times \dfrac{1}{3^{15}} \times \dfrac{1}{(-7)^{15}}$

(a)	-63	(b)	9
(c)	-55	(d)	80
(e)	0	(f)	-24

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(b) $(-5)^{-7 \times -2} = (-5)^{14}$

(c) $3^{13 \times -7} = 3^{-91}$

(d) $13^{-2 \times -4} = 13^{8}$

(e) $(-\sqrt{2})^{-5 \times -4} = (-\sqrt{2})^{20}$

(a)	17	(b)	5
(c)	0	(d)	18
(e)	-5	(f)	-10

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(b) $2^{14 - (-5)} = 2^{19}$

(c) $(-4)^{-6 - (-13)} = (-4)^{7}$

(d) $(\sqrt{2})^{-9 - (-4)} = (\sqrt{2})^{-5}$

(e) $3^{-18 - 3} = 3^{-21}$

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(b) $$\left(\frac{2}{5}\right)^{-3} = \frac{2^{-3}}{5^{-3}} = \frac{\frac{1}{2^3}}{\frac{1}{5^3}} = \frac{5^3}{2^3}$$

(c) $$\left(\frac{\sqrt{2}}{3}\right)^{-8} = \frac{(\sqrt{2})^{-8}}{3^{-8}} = \frac{\frac{1}{(\sqrt{2})^8}}{\frac{1}{3^8}} = \frac{3^8}{(\sqrt{2})^8}$$

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1. (a) reciprocal, $\frac{1}{3}$

   (b) 2, multiply, 4

   (c) 3, add, 8

   (d) 2

   (e) $\frac{1}{9}$

2. (b) $\frac{1}{7^4} = \frac{1}{2401}$

   (c) $\frac{1}{(-2)^5} = \frac{1}{-32}$

   (d) $\frac{1}{8^{-2}} = 8^2 = 64$

   (e) $\frac{8}{5}$

   (f) $5 \times 6^2 = 5 \times 36 = 180$

   (g) $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

   (h) $1 + 2^6 = 1 + 64 = 65$

   (i) $\frac{4^2}{2^3} = \frac{16}{8} = 2$

   (j) $\frac{2^{-4}}{3^{-4}} = \frac{3^4}{2^4} = \frac{81}{16}$

3. (b) $\frac{1}{5^{16}}$

   (c) 4^7

(d) $6 \times \frac{1}{8^{14}} = \frac{6}{8^{14}}$

(e) $\frac{1}{(-2)^{11}6^{2}}$

(f) $\frac{3^6}{4^{13}}$

(g) $\frac{2^8}{4^5 3^{12}}$

(h) $\frac{6^7}{2^{13}8^{10}}$

(i) $\frac{1}{5^{12}} + \frac{1}{6^8}$

4. (b) $7^{15 - (-8)} = 7^{23}$

(c) $\pi^{4 + 2 + 3} = \pi^9$

(d) $3^{-24} \times 3^{10} = 3^{-24 + 10} = 3^{-14}$.

(e) $\frac{2^{-5}}{3^{-5}} \times 2^6 \times 3^{-8} = 2^{-5 + 6}\, 3^{-8 - (-5)} = 2^1\, 3^{-3}$

(f) $\frac{6^9 \times 6^{-5}}{6^4} = 6^{9 - 5 - 4} = 6^0 = 1$

5. (b) $8^{-6 + 4} = 8^{-2} = \frac{1}{8^2} = \frac{1}{64}$

(c) $\frac{2^9 \times (-5)^9}{2^4 \times (-5)^{12}} = 2^{9 - 4}\,(-5)^{9 - 12} = 2^5\,(-5)^{-3} = \frac{2^5}{(-5)^3} = \frac{32}{-125}$

(d) $\frac{(-3)^{-24}}{(-3)^{-24}} = (-3)^{-24 - (-24)} = (-3)^{-24 + 24} = (-3)^0 = 1$

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What numeral does 10^5 represent? 100 000

What fraction does 10^{-15} represent? $\frac{1}{100\ 000}$

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1. 1 000 000	2. 100
3. $\frac{1}{100}$	4. 10 000
5. $\frac{1}{10}$	6. 100 000 000
7. 1	8. $\frac{1}{1000}$

1. 10^9	2. 10^{-6}
3. 10^0	4. 10^8
5. 10^{-3}	6. 10^{-7}

$$10 = 10^1, \frac{1}{10} = 10^{-1}, \frac{1}{100} = 10^{-2}$$

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$$130\,678.45 = (1 \times 10^5) + (3 \times 10^4) + (0 \times 10^3) + (6 \times 10^2) + (7 \times 10^1) + (8 \times 10^0) + (4 \times 10^{-1}) + (5 \times 10^{-2})$$

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3.4×10^{-5}	✓	13.5×10^2	___
2.0×10^8	✓	7.1×10^0	✓
0.1×10^{-3}	___	0.79×10^4	___
6.243×10^4	✓	0.439×10^{-2}	___
3.6×5^{10}	___	10×10^3	___

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Exercise – Using Powers of 10

1. (a) 74

   (b) 40 billion, 4×10^{10}

   (c) 3×10^{-13}

   (d) 0.000067

2. (a) true (b) false
   (c) true (d) false
   (e) true (f) true
   (g) false (h) false
   (i) true (j) false
   (k) false

3. (b) $\dfrac{1}{10\ 000}$ 10^{-4}

   (c) $\dfrac{1}{100}$ 10^{-2}

   (d) $\dfrac{1}{100\ 000\ 000}$ 10^{-8}

   (e) $\dfrac{1}{1\ 000\ 000}$ 10^{-6}

4. (a) $36.4057 = (3 \times 10^1) + (6 \times 10^0) + (4 \times 10^{-1}) + (0 \times 10^{-2}) + (5 \times 10^{-3}) + (7 \times 10^{-4})$

   (b) $68\ 432.9 = (6 \times 10^4) + (8 \times 10^3) + (4 \times 10^2) + (3 \times 10^1) + (2 \times 10^0) + (9 \times 10^{-1})$

5. (b) $10^{-5+3} = 10^{-2}$

   (c) $10^{-7-2} = 10^{-9}$

   (d) $10^{-6-(-4)} = 10^{-2}$

   (e) $10^{8-(-5)} = 10^{13}$

   (f) $10^{-2-4(-7)} = 10^{1}$

   (g) $10^{-8+3+2-(-3)-6} = 10^{-6}$

6. (b) 3000 3×10^3

(c) 0.000002 2×10^{-6}

(d) 90 000 000 9×10^7

(e) 0.06 6×10^{-2}

(f) 0.0006 6×10^{-4}

7. (b) 4 000 100.05

(c) 8.7056

(d) 0.00405

(e) 803 000.602

(f) 750.32

8. (b) 1400 (c) 777 000

(d) 4 000 000 000 (e) 0.0000066

(f) 1 090 000 (g) 0.006

(h) 20 000 (i) 120 000 000

(j) 0.000 000 5

9. (a) 3.4×10^7

(b) 5.6×10^{-5}

(c) 7.235×10^9

(d) 6×10^{-9}

10. (b) $\frac{1.6}{8} \times 10^{2-(-2)}$

$= 0.2 \times 10^4$

$= 2 \times 10^{-1} \times 10^4$

$= 2 \times 10^3$

(c) $= \dfrac{4 \times 9}{6} \times 10^{-9+5-(-3)}$

$= 6 \times 10^{-1}$

(d) $\dfrac{1.64 \times 2}{4} \times 10^{-4+13-7}$

$= 0.82 \times 10^{2}$

$= 8.2 \times 10^{-1} \times 10^{2}$

$= 8.2 \times 10^{1}$

11. (a) $(6.71 \times 10^{-4})(4.3 \times 10^{7})$

$= (6.71 \times 4.3)(10^{-4} \times 10^{7})$

$= 28.853 \times 10^{-4+7}$

$= 28.853 \times 10^{3}$

$= 28\ 853$

(b) $5.8 \times 10^{5} \times 6 \times 10^{8}$

$= 5.8 \times 6 \times 10^{5} \times 10^{8}$

$= 34.8 \times 10^{5+8}$

$= 34.8 \times 10^{13}$

$= 348\ 000\ 000\ 000\ 000$

(c) $\dfrac{1.83 \times 10^{5}}{3 \times 10^{-7}}$

$= 0.61 \times 10^{5-(-7)}$

$= 0.61 \times 10^{12}$

$= 610\ 000\ 000\ 000$

(d) $\dfrac{(2.4 \times 10^{-6})(5.5 \times 10^{4})}{(6 \times 10^{3})}$

$= \dfrac{2.4 \times 5.5}{6} \times \dfrac{10^{-6} \times 10^{4}}{10^{3}}$

$= 2.2 \times 10^{-6+4-3}$

$= 2.2 \times 10^{-5}$

$= 0.000022$

END OF LESSON 7

LESSON 8

Page 1

Variable Expression	Real Numbers	Variables	Arithmetic Operations
$\frac{2n^2}{3n} - 5p$	2, 3, 5	n, p	division, squaring, subtraction, multiplication
$\frac{3(a - b)}{7cd^3}$	3, 7	a, b, c, d	multiplication, subtraction, division, cubing
$9(a - 2b + 12c)$	9, 2, 12	a, b, c	multiplication, subtraction, addition
$5y - \sqrt{x + 2}$	5, 2	x, y	multiplication, subtraction, square root, addition

Page 2

1. divide
2. increase, multiply
3. square, -8
4. subtract, square
5. multiply, 7

Page 3

(a) 2
(b) -6
(c) $25 - 16 = 9$
$= 25 - 20$
$= 5$

Page 5

...real number **-3** and the variable **a**. The second term is the quotient of the variable **b** and the real number **5**. The third term is the variable **c**.

Name the terms in the variable expression $\frac{2a}{3} - 5b + c - 8d^2$.

$$\mathbf{\frac{2a}{3}, -5b, +c, -8d^2}$$

Write a variable expression that has five terms.

a + b + c + d + e (or any other similar example)

Name the five terms.

a, b, c, d, e

Page 6

In the above example, the first factor is the real number **7**. The second factor is the sum of the variable **x** and the real number **3**. The third factor is the difference of the variable **y** and the real number **5**.

$x,\ y + z,\ y^2 - 3z$

Page 7

$5 \times (a - b) \times (a - b)$

$6 \times m \times m \times n \times n$

$(x + y)\ (x + y)\ (x - y)\ (x - y)$

$2 \times x \times y \times y \times z \times z \times z$

...factor? **3**
...factor? **1**

Name the factors of the variable expression $\frac{3mn}{r}$.

$\mathbf{3, m, n, \frac{1}{r}}$

Page 10

1. (a) domain (b) -6

   (c) subtract, multiply, $\frac{2}{3}$ (d) three

   (e) variable (f) -3, $(x + 2)$, $(x - 7)$, $(x - 7)$

   (g) like, literal (h) numerical, literal

   (i) factors (j) terms

   (k) -1

2. (a) ✓ (b) ___
   (c) ✓ (d) ✓
   (e) ___ (f) ✓
   (g) ✓ h) ___
   (i) ✓

3. (a) (ii) $= 3(0)^2 - 0 - 4 = -4$

(iii) $= 3(2)^2 - 2 - 4 = 6$

(b) (i) $= (-2)^3 + 3(-2) = -8 - 6 = -14$

(ii) when $x = -1$, $x^3 + 3x = (-1)^3 + 3(-1) = -1 - 3 = -4$

(iii) when $x = 3$, $x^3 + 3x = 3^3 + 3(3) = 27 + 9 = 36$

(c) (i) $\dfrac{-6 + 6}{-6 - 5} = \dfrac{0}{-11} = 0$

(ii) when $t = -3$, $\dfrac{t + 6}{t - 5} = \dfrac{-3 + 6}{-3 - 5} = \dfrac{3}{-8} = \dfrac{-3}{8}$

(iii) when $t = 0$, $\dfrac{t + 6}{t - 5} = \dfrac{0 + 6}{0 - 5} = \dfrac{6}{-5} = \dfrac{-6}{5}$

(iv) when $t = 5$, $\dfrac{t + 6}{t - 5} = \dfrac{5 + 6}{5 - 5} = \dfrac{11}{0} =$ undefined

4. (b) $-8x\ (a + b) \times (a + b) \times c \times c \times c$

(c) $(x - y) \times (x - y) \times z \times z \times \frac{1}{y} \times \frac{1}{y}$

(d) $4 \times m \times n \times t \times t$

(e) $(x + y) \times (x + y) \times (x + y) \times z \times z \times z \times \frac{1}{3} \times \frac{1}{y}$

5. (a)
$= 8 - (5 + 2)$
$= 8 - 7$
$= 1$

(b)
$(3 \times -1 \times 2)^3$
$= (-6)^3$
$= -6 \times -6 \times -6$
$= -216$

(c)
$\dfrac{-4}{8} + \dfrac{5}{2}$

$= \dfrac{-1}{2} + \dfrac{5}{2}$

$= \dfrac{4}{2}$

$= 2$

(d)
$5(2)^2(-3) + 3(-3)(4)$

$= 5(4)(-3) + 3(-12)$

$= -60 + -36$

$= -96$

(e) $-3(5 + -7)^2$
$= -3\,(-2)^2$
$= -3(4)$
$= -12$

(f) $(-2)(3) - (3)(4) + 5(-2)(4)$
$= -6 - 12 - 40$
$= - 58$

Page 13

Expression	Think Step	Sum
$2x + 4x + 11x$	$(2 + 4 + 11)\,x$	$17x$
$7y + y + 9y$	$(7 + 1 + 9)\,y$	$17y$
$2x^2y + 4x^2y + 9x^2y$	$(2 + 4 + 9)\,x^2y$	$15x^2y$
$2ab + ab$	$(2 + 1)ab$	$3ab$
$a^2 + a^2 + a^2$	$(1 + 1 + 1)\,a^2$	$3a^2$
$\frac{1}{2}b + \frac{1}{2}b$	$(\frac{1}{2} + \frac{1}{2})\,b$	$1b$
$3.5a^2b^2 + 0.5a^2b^2$	$(3.5 + 0.5)a^2b^2$	$4a^2b^2$

Page 14

1. $= (3a^2 + a^2 + 7a^2) + (2a + 5a) + 4$

   $= 11a^2 + 7a + 4$

2. $(4x + 5x) + (3y + 9y)$

   $= 9x + 12y$

3. $(-2ab + 5ab) + (3b^2 + 4b^2) + (2a + 5a)$

   $= 3ab + 7b^2 + 7a$

4. $(2x^3 + 9x^3) + 5x^2 + (3x + 7x)$

   $= 11x^3 + 5x^3 + 10x$

Page 15

Expression	Think Step	Difference
$11a^2 - a^2$	$(11 - 1)\,a^2$	$10a^2$
$6mn - 10mn$	$(6 - 10)mn$	$-4mn$
$6x^2y - 7x^2y$	$(6 - 7)x^2y$	$-x^2y$
$-3r - 5r$	$(-3 - 5)r$	$-8r$

Page 16

1. $(3a^2 - 4a^2 - 6a^2) + (-12ab + 7ab + 3ab) + (2b - 9b)$

   $= -7a^2 - 2ab - 7b$

2. $(x^2 - 3x^2 + 4x^2) + (4x - 9x + 8x) + (-9 + 3 - 7)$

   $= 2x^2 + 3x - 13$

3. $(-3m^2 + 3m^2) + (-n^2 - 5n^2 + 3n^2) + (4mn - 8mn - 5mn)$

   $= -3n^2 - 9mn$

Page 17

1. $a - b + c$
2. $-x + y - z$
3. $-x^2 + xy - y^2 + x^2y^2$

Page 20

1. $z^{5+2} = z^7$
2. $m^{8+(-6)} = m^2$
3. $c^{-4+-3} = c^{-7}$
4. $x^{2+2} = x^4$
5. $y^{4+5+1} = y^{10}$

Page 21

1. $-6bc$

2. $(-6 \times -4)\ (x \times x) = 24x^{1+1} = 24x^2$

3. $(3 \times -8)\ (a^2 \times a^2)\ (b^2 \times b^3) = -24a^{2+1}\ b^{2+3} = -24a^3b^5$

4. $(6 \times 5)(y^3xy^{-8}) = 30y^{3-8} = 30y^{-5}$

5. $(-9 \times \frac{1}{3})\ (x^3 \times x)\ (y^2 \times y^2)\ (z^5 \times z^{-3}) = -3x^4y^4z^2$

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1. $(4x^2)(5x^3) + (4x^2)(3x^2) + (4x^2)(-5x)$

   $= 20x^5 + 12x^4 - 20x^3$

2. $(ab)(a^2) + (ab)(-b^2) + (ab)(5b) + (ab)(-9ab)$

   $= a^3b - ab^3 + 5ab^2 - 9a^2b^2$

3. $(-3x)(x^2) + (-3x)(-2x) + (-3x)(1)$

   $= -3x^3 + 6x^2 - 3x$

Page 23

1. $15a^2 + 20ab + 3ab + 4b^2$

   $= 15a^2 + 23ab + 4b^2$

2. $(2x)(2x) + (-3y)(2x) + (2x)(3y) + (-3y)(3y)$

   $= 4x^2 - 6xy + 6xy - 9y^2$

   $= 4x^2 - 9y^2$

3. $(2m)(3r) + (-n)(3r) + (2m)(s) + (-n)(s)$

   $= 6mr - 3nr + 2\ ms - ns$

Page 25

1. $\dfrac{1}{z^{5-2}} = \dfrac{1}{z^3}$

2. $a^{8-4} = a^4$

3. $x^{4-4} = x^0 = 1$

4. $m^{3-(-3)} = m^6$

5. $y^{4-3} = y^1 = y$

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1. $5 \times x^{3-2} = 5x$

2. $\dfrac{4}{16} \times \dfrac{y^4}{y^9} = \dfrac{1}{4} \times \dfrac{1}{y^{9-4}} = \dfrac{1}{4y^5}$

3. $\dfrac{9}{3} \times \dfrac{z^5}{z} = 3 \times z^{5-1} = 3z^4$

4. $\dfrac{-18}{16} \times \dfrac{r^7}{r^3} \times \dfrac{s^3}{s^5} \times \dfrac{t^5}{t^3} = \dfrac{-9}{8} \times \dfrac{r^{7-3}}{s^{5-3}} \times t^{5-3} = \dfrac{-9\ r^4t^2}{8\ s^2}$

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Exercise – Operating With Variable Expressions

1. (a) distributive

   (b) communtative, associative

   (c) quotient of powers

   (d) add, additive

   (e) The sum of a^3 and a^3 is $\mathbf{2a^3}$.

   The difference of a^3 and a^3 is **0**.

   The product of a^3 and a^3 is $\mathbf{a^6}$.

   The quotient of a^3 and a^3 is **1**.

(f) like, distributive, numerical

(g) additive inverse, $-x + y + 2z$

(h) terms

(i) factors

2. (a) $13y$
   (b) $-x$
   (c) $-2ab$
   (d) $-10m$
   (e) 0
   (f) $5r$
   (g) $14x$
   (h) $4m - 2n$
   (i) $3x + y - 3z$
   (j) $-4x^2y + 8x^2$
   (k) $-4ab + 4b^2 - 3ab^2$
   (l) $2x^2 - 3xy$
   (m) $(r^2 + r + 6) + (-2r^2 + 2r + 5)$
   $= (r^2 - 2r^2) + (r + 2r) + (6 + 5)$
   $= -r^2 + 3r + 11$
   (n) $(2x^2 + 3x - 5) + (x^2 - 5x + 7) + (-3x^2 - 2x - 1)$
   $= (2x^2 + x^2 - 3x^2) + (3x - 5x - 2x) + (-5 + 7 - 1)$
   $= 0x^2 - 4x + 1)$
   $= -4x + 1$
   (o) $(a - 2b) - (a + b) + (2a - 5b) - (-a - b)$
   $= a - 2b - a - b + 2a - 5b + a + b$
   $= (a - a + 2a + a) + (-2b - b - 5b + b)$
   $= 3a - 7b$

3. (a) y^{17}
   (b) $-x^5$
   (c) $6x^3y$
   (d) $8a^2b$
   (e) y^{-1}
   (f) $4ab^4$
   (g) $-5x^{-2}y^{-2}$
   (h) $6a^3b^3c^3$
   (i) $-42x^5y^5$
   (j) $-\frac{1}{4}y^4z^5$
   (k) $-4m^3n^4$
   (l) $3a^2b^3$
   (m) $4x^2 + 4xy$
   (n) $3ab - 3b^2$

(o) $-2m^3 + 6m^2n - 2mn^2$ (p) $-8x^4y + 12x^3y^3$

(q) $-2a - 2b + 2c - 2d$ (r) $6a^3bc - 12ab^2c^2$

(s) $-x + y - z$ (t) $-3y^2 + 6yz$

(u) $a^2 - 2a - 15$ (v) $12x^2 - 28x + 3x - 7$
$= 12x^2 - 25x - 7$

(w) $6a^2 + 2ab - 15ab - 5b^2$
$= 6a^2 - 13ab - 5b^2$ (x) $x^2y^2 + 5xyz - 5xyz - 25z^2$
$= x^2y^2 - 25z^2$

(y) $-y^2 + yz + yz - z^2$
$= -y^2 + 2yz - z^2$ (z) $6x^2 - 9x + 8x - 12$
$= 6x^2 - x - 12$

4. (a) y (b) $\dfrac{1}{x^2}$

(c) $-5m^4$ (d) $\dfrac{-y^3}{4}$

(e) $\dfrac{x^2y^2}{2}$ (f) $-2a$

(g) $\dfrac{3}{m}$ (h) $-5c$

(i) $-\dfrac{1}{3} f^{7-1} g^{2-0} = -\dfrac{1}{3} f^6 g^{-7} = \dfrac{-f^6}{3g^7}$

(j) $\dfrac{-1}{5m^3n^5}$ (k) $\dfrac{4x}{3y^6}$

(l) $7x^2y^2z$ (m) $-4a$

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$(-\frac{1}{2})^5$, $(3)^3$, $(3a)^2$, $(5x)^5$, or any other similar answer.

$(\sqrt{2})^4$ means $\sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2}$

$(\frac{-1}{2})^5$ means $\frac{-1}{2} \times \frac{-1}{2} \times \frac{-1}{2} \times \frac{-1}{2} \times \frac{-1}{2}$

π^2 means $\pi \times \pi$

$(-2a)^5$ means $-2a \times -2a \times -2a \times -2a \times -2a$

$(\frac{2a}{b})^4$ means $\frac{2a}{b} \times \frac{2a}{b} \times \frac{2a}{b} \times \frac{2a}{b}$

$(-x^2y)^3$ means $-x^2y \times -x^2y \times -x^2y$

$(\frac{3}{4})^{-5}$, $(3)^{-2}$, $(6a^2)^{-3}$, $(a)^{-6}$, or any other similar answer.

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1. $\dfrac{1}{y^5}$

2. $\dfrac{6}{\frac{1}{x^4}} = \dfrac{6}{1} \div \dfrac{1}{x^4} = \dfrac{6}{1} \times \dfrac{x^4}{1} = 6x^4$

3. $\dfrac{4}{1} \times \dfrac{1}{x^2} \times \dfrac{y^3}{1} = \dfrac{4y^3}{x^2}$

Page 37

1. $y^{-3 \times 8} = y^{-24}$

2. $\dfrac{1}{x^{-5}x^4} = \dfrac{1}{x^{-20}}$

3. m^6

4. $5a^{2 \times 3} = 5a^6$

5. $(-r)^{-4 \times 2} = (-r)^{-8}$

1. $(-4)^3a^3b^3 = -64a^3b^3$

2. $9^2x^2 = 81x^2$

3. $2^5x^5y^5z^5 = 32x^5y^5z^5$

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1. $\dfrac{3^4}{x^4} = \dfrac{81}{x^4}$

2. $\dfrac{(-c)^7}{b^7}$

3. $\dfrac{(xy)^6}{z^6}$

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$$(-3a^3b^2)^4 = (-3)^4(a^3)^4(b^2)^4 = 81a^{3\times 4}b^{2\times 4} = 81a^{12}b^8$$

$$\left(\frac{-5x^3}{7z^2}\right)^2 = \frac{(-5x^3)^2}{(7z^2)^2} = \frac{(-5)^2(x^3)^2}{7^2(z^2)^2} = \frac{25x^{3\times 2}}{49z^{2\times 2}} = \frac{25x^6}{49z^4}$$

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Exercise – Powers of Variable Terms

1. (a) (a + b), five

   (b) product, powers

   (c) x^2y^2

   (d) y^4, $\frac{1}{y^4}$

   (e) x^2

2.

D	1.	B	6.
E	2.	C	7.
F	3.	D	8.
C	4.	A	9.
D	5.	E	10.

3. (b) $a^4b^5c^3$

   (c) $\frac{5x^2y^2}{7z^3}$

   (d) $\frac{t^6}{5^2} - r^3$

   (e) $\frac{4d^2}{c^2}$

4. (a) $16y^2$

   (b) $-8b^3$

   (c) $-125x^6y^3$

   (d) $81x^8y^4$

   (e) $81x^{16}y^{12}z^4$

   (f) $\frac{4x^2}{y^2}$

   (g) $\frac{27}{8b^3}$

   (h) $\frac{-243m^{10}}{n^{15}}$

   (i) $\frac{64a^6b^{12}}{c^{12}}$

5. (a) $a^{-2}b^{-7}c^{-7}$

(b) $\dfrac{a^5b^5}{a^2b^3} = a^{5-2}b^{5-3} = a^3b^2$

(c) $9x^2y^2 - 12x^2y^2 = (9 - 12)x^2y^2 = -3x^2y^2$

(d) $\dfrac{x^{10}}{x^6} = x^{10-6} = x^4$

(e) $\dfrac{x^{13}}{x^4 \times x^6} = x^{13-4-6} = x^3$

(f) $\dfrac{x^{-3}}{y^{-3}} \times \dfrac{y^5}{x} = x^{-3-1}\, y^{5-(-3)} = x^{-4}\, y^8 = \dfrac{y^8}{x^4}$

(g) $a^6 \times a^{-8} = a^{6-8} = a^{-2}$

6. (a) $\dfrac{3}{x^5}$

(b) $9x^2$

(c) $\dfrac{x^2y^5}{4}$

(d) $\dfrac{1}{a^3} + \dfrac{1}{b^3}$

(e) $\dfrac{10d}{c}$

(f) $\dfrac{4n}{m^3}$

(g) $\dfrac{5a^2}{b}$

(h) $\dfrac{3y^3}{2x^5}$

(i) $\dfrac{3a^2}{b^4c}$

END OF LESSON 8

LESSON 9

Page 1

1. $r + s$

2. $x - 2$

Page 2

3. $-6y$

4. $\frac{m}{2}$

Exercise – Mathematical Phrases

1. (b) addition
   (c) subtraction
   (d) division
   (e) division
   (f) subtraction
   (g) addition
   (h) multiplication
   (i) addition

2. (b) $\frac{n}{2}$

   (c) $2n$

   (d) $n - 7$

   (e) $5n$

   (f) $7n$

   (g) $\frac{n}{6}$

   (h) $8 - n$

   (i) $n + 3$

   (j) $\frac{5}{n}$

   (k) $n - 8$

Page 4

1. (b)

Take a number	n
Twice the number	$2n$
Add 9	$2n + 9$

(c)

Take a number	n
Multiply by 4	$4n$
Subtract 5	$4n - 5$

(d)

Take a number	n
Divide by 4	$\frac{n}{4}$
Add 2	$\frac{n}{2} + 2$

(e)

Take a number	n
Divide by 2	$\frac{1}{2}n$
Add 5	$\frac{1}{2}n + 5$

(f)

Take a number	n
Multiply by 3	$3n$
Divide by 4	$\frac{3n}{4}$

2. (a) $5n - 8$

(b) $6 \div 3n$

(c) $6n + 18$

(d) $\frac{2}{3}n - 5$

(e) $8 + \frac{3}{n}$

(f) $\frac{4 + n}{9}$

Page 6

1. (a) Divide by 4

   (b) Subtract 7
   Divide by 5

   (c) Add 3

   (d) Subtract 1
   Multiply by 5

2. (a) y

   (b) 2x
   x

   (c) 3y
   y

   (d) 2x
   x

3. (a) 3x
   x

   (b) Add 9 — $\frac{1}{2}y$

   Multiply by 2 — y

   (c) Multiply by 5 — 3n
   Divide by 3 — n

   (d) Subtract 10 — 3y
   Divide by 3 — y

   (e) Subtract 5 — $\frac{x}{7}$

   Multiply by 7 — x

Page 9

Give examples of three other number equations that are true.

$\frac{1}{2} \times \frac{-6}{1} = -3$ $\qquad$ $\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$ $\qquad$ $\frac{3}{4} \times \frac{1}{5} = \frac{3}{20}$

(Or any other similar example.)

Give examples of three other number equations that are false.

$\frac{1}{2} \times \frac{1}{3} = \frac{1}{5}$ $\qquad$ $\frac{2}{5} \times \frac{1}{3} = \frac{2}{8}$ $\qquad$ $\frac{1}{4} \times \frac{1}{3} = \frac{1}{7}$

(Or any other similar examples.)

Decide whether each of the following number equations is true or false.

$3 \times 5 = 5 \times 3$	$13 \times 0 = 13$	$(6 \div 2) + 5 = (2 \div 6) + 5$
true	false	false
$(9 \times 1) + 3 = 13$	$4 + 4 + 4 = 3 \times 4$	$36 = (3 \times 10) + 6$
false	true	true

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Exercise – Mathematical Equations

1. (a) true $\qquad$ (b) conditional
   (c) false $\qquad$ (d) conditional
   (e) conditional $\qquad$ (f) true

2. (b)
$$\begin{aligned} 7(5) + 12 &= 54 \\ 35 + 12 &= 54 \\ 47 &= 54 \end{aligned}$$
false

   (c)
$$\begin{aligned} 3(2)(3) - 2 &= 7 \\ 18 - 2 &= 7 \\ 16 &= 7 \end{aligned}$$
false

   (d)
$$\begin{aligned} (-3)^2 + 8(-3) &= -15 \\ 9 - 24 &= -15 \\ -15 &= -15 \end{aligned}$$
true

   (e)
$$\begin{aligned} (-5)^2 + 8(-5) &= -15 \\ 25 - 40 &= -15 \\ -15 &= -15 \end{aligned}$$
true

(f) $5(-1)^2 + (-1)(-3) = 8$
$5 + 3 = 8$
$8 = 8$
true

(g) $6 + 2(-5) = 4$
$6 + -10 = 4$
$-4 = 4$
false

(h) $2(2) - 3(-1) = 7$
$4 + 3 = 7$
$7 = 7$
true

(i) $2(2) + 3 = -5(2) + 17$
$4 + 3 = -10 + 17$
$7 = 7$
true

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$2(-3) - 5 = 11$	$2(5) - 5 = 11$	$2(8) - 5 = 11$
$-6 - 5 = 11$	$10 - 5 = 11$	$16 - 5 = 11$
$-11 = 11$	$5 = 11$	$11 = 11$
false	false	true

Since we obtain a true statement when $n = \mathbf{8}$, we say that **8** is a root of the equation and the solution set of the equation is **8** for the given domain.

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2. 5, 5
3. 8, 8
4. 5, 5
5. 5, 5
6. 28, 28
7. 15, 15
8. 3, 3
9. −2, −2
10. 4, 4

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1. $x - 9 + 9 = 12 + 9$ $x = 21$	$x - 1 + 1 = 7 + 1$ $x = 8$	$x - 4 + 4 = 3 + 4$ $x = 7$
2. $x + 9 - 9 = 12 - 9$ $x = 3$	$x + 4 - 4 = 23 - 4$ $x = 19$	$x + 3 - 3 = 1 - 3$ $x = -2$
3. $7 \times \frac{y}{7} = 7 \times 4$ $y = 28$	$2 \times \frac{1}{2}x = 2 \times 7$ $x = 14$	$5 \times \frac{y}{5} = 5 \times (-2)$ $y = -10$

4. $\frac{3x}{3} = \frac{21}{3}$ $\quad$ $\frac{-7x}{-7} = \frac{28}{-7}$ $\quad$ $\frac{4x}{4} = \frac{24}{4}$

$x = 7$ $\quad$ $x = -4$ $\quad$ $x = 6$

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Exercise – Solving Equations

1. (b) Add 3 to each side
   (c) Multiply each side by 6
   (d) Divide each side by 5
   (e) Subtract 4 from each side
   (f) Multiply each side by 3
   (h) Subtract 9 from each side
   (i) Add 6 to each side

2. (a) $x = 13$

   (b) $7 \times \frac{x}{7} = 7 \times 3$
   $x = 21$

   (c) $x + 5 - 5 = 1 - 5$
   $x = -4$

   (d) $\frac{-2x}{2} = \frac{14}{-2}$
   $x = -7$

   (e) $\frac{-6x}{-6} = \frac{-12}{-6}$
   $x = 2$

   (f) $4 \times \frac{1}{4}x = -3 \times 4$
   $x = -12$

   (g) $x - 12 + 12 = 8 + 12$
   $x = 20$

   (h) $x + 7 - 7 = 13 - 7$
   $x = 6$

   (i) $10 \times \frac{x}{10} = 11 \times 10$
   $x = 110$

   (j) $x - 9 + 9 = -3 + 9$
   $x = 6$

   (k) $x - 12 + 12 = 13 + 12$
   $x = 25$

   (l) $\frac{-3x}{-3} = \frac{27}{-3}$
   $x = -9$

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Exercise – Solving Equations

1. (a) Multiply each side by 3
   Divide each side by 2

   (b) Add 1 to each side
   Multiply each side by 3

   (c) Subtract 3 from each side
   Divide each side by -2

   (d) Add 2 to each side
   Multiply each side by 4

   (e) Add 4x to each side
   Subtract 20 from each side
   Multiply each side by -1

   (f) Subtract 5x from each side
   Subtract 6 from each side
   Divide each side by -2

   (g) Use distributive property
   Subtract 6x from each side
   Subtract 30 from each side
   Multiply each side by -1

   (h) Multiply each side by 30
   Use distributive property
   Collect like terms
   Subtract 2x from each side
   Subtract 9 from each side
   Divide each side by -5

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2. (b) $\frac{x}{8} = 5$

   $x = 40$

   Check: $\frac{x}{8} - 3 = 2$

   $$\begin{array}{l|l} \frac{40}{8} - 3 & 2 \\ 5 - 3 & \\ 2 & \end{array}$$

   (c) $6x = 6$
   $x = 3$

   Check: $2x = 6 - 4x$

   $$\begin{array}{l|l} 2(1) & 6 - 4(1) \\ 2 & 2 \end{array}$$

   (d) $6x = 18$
   $x = 3$

   Check: $8x - 5 = 2x + 13$

   $$\begin{array}{l|l} 8(3) - 5 & 2(3) + 13 \\ 24 - 5 & 6 + 13 \\ 19 & 19 \end{array}$$

   (e) $-x = 6$
   $x = -6$

   Check: $4 - 3x = 10 - 2x$

   $$\begin{array}{l|l} 4 - 3(-6) & 10 - 2(-6) \\ 4 + 18 & 10 + 12 \\ 22 & 22 \end{array}$$

(f) $8m - 5 = 31 - m$

$9m = 36$

$m = 4$

Check:

$6m - 5 + 2m = 20 - m + 11$

$6(4) - 5 + 2(4)$	$20 - 4 + 11$
$24 - 5 + 8$	27
27	

(g) $5n = -180$

$n = -36$

Check:

$\frac{5n}{3} = -60$

$\frac{5(36)}{3}$	-60
	-60

(h) $7x - (2 + x) = 2(4x - 2)$

$7x - 2 - x = 8x - 4$

$6x - 2 = 8x - 4$

$-2x = -2$

$x = 1$

Check:

$7x - (2 + x) = 2(4x - 2)$

$7(1) - (2 + 1)$	$2(4 - 2)$
$7 - 3$	$2(2)$
4	4

(i) $5x = 4x + 14$

$x = 14$

Check:

$\frac{x}{4} = \frac{x}{5} + \frac{7}{10}$

$\frac{14}{4}$	$\frac{14}{5} + \frac{7}{10}$
$\frac{7}{2}$	$\frac{28}{10} + \frac{7}{10}$
$\frac{7}{2}$	$\frac{35}{10}$
$\frac{7}{2}$	$\frac{7}{2}$

(j) $2x + 2 - 3x + 3 = 3x - 3 - 3x + 1$

$-x + 5 = -2$

$-x = -7$

$x = 7$

$\frac{x + 1}{3} - \frac{x - 1}{2} = \frac{x - 1}{2} - \frac{3x - 1}{6}$

$\frac{7 + 1}{3} - \frac{7 - 1}{2}$	$\frac{7 - 1}{2} - \frac{3(7) - 1}{6}$
$\frac{8}{3} - \frac{6}{2}$	$\frac{6}{2} - \frac{20}{6}$
$\frac{8}{3} - 3$	$3 - \frac{10}{3}$
$\frac{8}{3} - \frac{9}{3}$	$\frac{9}{3} - \frac{10}{3}$
$\frac{-1}{3}$	$\frac{-1}{3}$

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2. $5n - 5 = 20$
   $5n = 25$
   $n = 5$

3. $n + 3n = 28$
   $4n = 28$
   $n = 7$

4. $\frac{1}{2}n + 7 = 11$

   $\frac{1}{2}n = 4$

   $n = 8$

5. $8 - 3n = 2$
   $-3n = -6$
   $n = 2$

6. $\frac{n}{2} + 3 = 9$

   $\frac{n}{2} = 6$

   $n = 12$

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2. $x + 3x = 84$
   $4x = 84$
   $x = 21$
   $3x = 63$

   The numbers are 21 and 63.

3. Then $(2n - 7)$ is the number of girls.

   $n + (2n - 7) = 29$
   $3n - 7 = 29$
   $3n = 36$
   $n = 12$
   $2n - 7 = 17$

   There were 12 boys and 17 girls in the class.

4. Then $(y + 7)$ is the amount he earned the first week, and $2y is the amount he earned in the third week.

   $y + (y + 7) + 2y = 51$
   $4y + 7 = 51$
   $4y = 44$
   $y = 11 \longleftarrow$ second week
   Thus, $y + 7 = 18 \longleftarrow$ first week
   $2y = 22 \longleftarrow$ third week

   Mike earned \$18, \$11, and \$22 in the three weeks.

5. Then the length is $(x + 7)$ cm.

$$\begin{aligned} 2x + 2(x + 7) &= 54 \\ 2x + 2x + 14 &= 54 \\ 4x &= 40 \\ x &= 10 \longleftarrow \text{width} \\ x + 7 &= 17 \longleftarrow \text{length} \end{aligned}$$

The dimensions are 10cm × 17 cm.

6. Five times the number, decreased by 8, is $(5n - 8)$. The number increased by 8 is $\underline{n + 8}$.

$$\begin{aligned} 5n - 8 &= n + 8 \\ 4n - 8 &= 8 \\ 4n &= 16 \\ n &= 4 \end{aligned}$$

The number is 4.

7. Then Don invested $\$\underline{2x}$ and Time invested $\$\underline{x + 16}$.

$$\begin{aligned} x + 2x + x + 16 &= 96 \\ 4x + 16 &= 96 \\ 4x &= 80 \\ x &= 20 \\ 2x &= 40 \\ x + 16 &= 36 \end{aligned}$$

Jack invested \$20, Don \$40, and Tim \$36.

8. Then $(x + 10)°$ is the measure of the second angle and $(x + 20)°$ is the measure of the third angle.

$$\begin{aligned} x + (x + 10) + (x + 20) &= 180 \\ 3x + 30 &= 180 \\ 3x &= 150 \\ x &= 50 \\ x + 10 &= 60 \\ x + 20 &= 70 \end{aligned}$$

The measurements of the angles are 50°, 60°, and 70°.

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Give examples of three other number inequalities that are true.

$\underline{5 - 4 < 0 + 2}$ $\underline{6 - 2 < 7 - 1}$ $\underline{4 - 2 > 3 - 7}$

(or any other similar example)

Give examples of three other number inequalities that are false.

$\underline{3 + 6 < 5 - 2}$ $\underline{2 + 1 > 8 - 4}$ $\underline{2 + 1 < 5 - 6}$

(or any other similar example)

$-2 \times 3 < 5$

true

$-2 > -1$

false

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$3 \times 6 < 9 + 2$

false

$5 + 7 > 15 - 5$

true

$2 + 3 \leqq 7$

true

Exercise – Inequalities

1. (b) $2n + 1 > 7$

   (c) $\frac{n}{7} - 5 \geqq 41$

   (d) $\frac{n}{3} \leqq 99$

   (e) $n + 5n < 60$

2. (a) true
   (b) false
   (c) conditional
   (d) conditional
   (e) true
   (f) conditional

3. (b) $3(0) + 5 \geqq 6$

   $0 + 5 \geqq 6$

   $5 \geqq 6$

   false

   (c) $2(2)(-1)-3 \leqq -4$

   $-4 - 3 \leqq -4$

   $-7 \leqq -4$

   true

   (d) $(-2)^2 + 1 \leqq 0$

   $4 + 1 \leqq 0$

   $5 \leqq 0$

   false

   (e) $4 + 1 \geqq 2 + 3$

   $5 \geqq 5$

   true

   (f) $3(2) + 2(3) \geqq 11$

   $6 + 6 \geqq 11$

   $12 \geqq 11$

   true

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If $x = 1$

$$\begin{aligned} 2x - 5 &\leqq 1 \\ 2(1) - 5 &\leqq 1 \\ 2 - 5 &\leqq 1 \\ -3 &\leqq 1 \end{aligned}$$

true

If $x = 2$

$$\begin{aligned} 2x - 5 &\leqq 1 \\ 2(2) - 5 &\leqq 1 \\ 4 - 5 &\leqq 1 \\ -1 &\leqq 1 \end{aligned}$$

true

If $x = 3$

$$\begin{aligned} 2x - 5 &\leqq 1 \\ 2(3) - 5 &\leqq 1 \\ 6 - 5 &\leqq 1 \\ 1 &\leqq 1 \end{aligned}$$

true

If $x = 4$

$$\begin{aligned} 2x - 5 &\leqq 1 \\ 2(4) - 5 &\leqq 1 \\ 8 - 5 &\leqq 1 \\ 3 &\leqq 1 \end{aligned}$$

false

If $x = 5$

$$\begin{aligned} 2x - 5 &\leqq 1 \\ 2(5) - 5 &\leqq 1 \\ 10 - 5 &\leqq 1 \\ 5 &\leqq 1 \end{aligned}$$

false

We obtain a true statement when $x =$ **0**, **1**, **2**, **and** **3**. Thus, the solution set of the inequality is **{0, 1, 2, 3}...**

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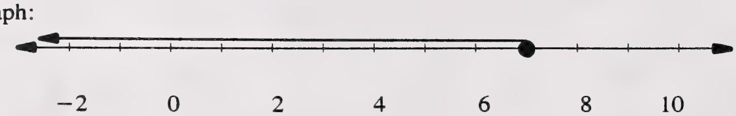
1. This inequality is satisfied when $x >$ 2 since it becomes a true statement whenever any real number greater than 2 is substituted for x. The solution set is **$\{x \mid x < 2, x \in R\}$**

Graph:

2. This inequality is satisfied when $x \leqq$ 7 since it becomes a true statement whenever any real number less than or equal to 7 is substituted for x. The solution set is **$\{x \mid x \leqq 7, x \in R\}$**

Graph:



3. This inequality is satisfied when $x \geqq 2$ since it becomes a true statement whenever any real number greater than or equal to 2 is subsituted for x. The solution set is $\{x \mid x \geqq 2, x \in \mathbf{R}\}$

Graph:

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1. $x - 5 + 5 \geqq -2 + 5$ $\quad$ $x - 1 + 1 < 9 + 1$ $\quad$ $x - 3 + 3 > -7 + 3$

$x \geqq 3$ $\quad$ $x < 10$ $\quad$ $x > -4$

2. $x + 5 - 5 \geqq 12 - 5$ $\quad$ $x + 1 - 1 < -3 - 1$ $\quad$ $x + 6 - 6 > 2 - 6$

$x \geqq 7$ $\quad$ $x < -4$ $\quad$ $x > -4$

3. $6 \times \frac{x}{6} > 6 \times 3$ $\quad$ $3 \times \frac{1}{3}x < 3 \times \frac{3}{4}$ $\quad$ $2 \times \frac{x}{2} \geqq 2x - 1$

$x > 18$ $\quad$ $x < \frac{9}{4}$ $\quad$ $x \geqq -2$

4. $\frac{4x}{4} \geqq \frac{32}{4}$ $\quad$ $\frac{6x}{6} < \frac{3}{6}$ $\quad$ $\frac{2x}{2} > \frac{-1}{2}$

$x \geqq 8$ $\quad$ $x < \frac{1}{2}$ $\quad$ $x > \frac{-1}{2}$

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$\frac{-7x}{-7} > \frac{14}{-7}$ $\quad$ $\frac{-2x}{-2} < \frac{16}{-2}$ $\quad$ $\frac{-4x}{-4} \leqq \frac{-24}{-4}$

$x > -2$ $\quad$ $x < -8$ $\quad$ $x \leqq 6$

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$-1 \times -x < -1 \times 5$ $\quad$ $-2 \times \frac{-1}{2}x \geqq -2x - 3$ $\quad$ $-9 \times \frac{x}{-9} \leqq -9 \times 1$

$x < -5$ $\quad$ $x \geqq 6$ $\quad$ $x \leqq -9$

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Exercise – Linear Inequalities in One Variable

1. (a) equivalent
   (b) divide, -2, direction
   (c) $\{9, 10\}$
   (d) linear, one
   (e) greater, 6
   (f) $\{x \mid x > -3, x \in R\}$
   (g) negative

2. (a) Subtract 5 from each side

   (b) Add 7 to each side
   Multiply each side by 2

   (c) Multiply each side by 12
   Subtract 6x from each side
   Add 9 to each side
   Divide each side by 2

   (d) Subtract 5x from each side
   Subtract 4 from each side
   Divide each side by -2 and change arrow

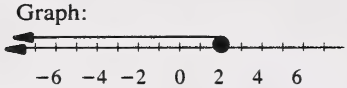
   (e) Multiply each side by -2 and change arrow
   Subtract 5 from each side
   Divide each side by 3

   (f) Use distributive property
   Subtract 5x from each side
   Subtract 15 from each side
   Divide each side by -8 and change arrow

3. (a)
$$\begin{aligned} 3x + 4 &\leqq 10 \\ 3x &\leqq c \\ x &\leqq 2 \end{aligned}$$

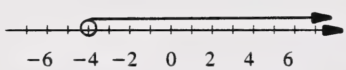
Solution set is $\{x \mid x \leqq 2, x \in R\}$

Graph:



(b)
$$\begin{aligned} -5x &< 20 \\ x &> -4 \end{aligned}$$

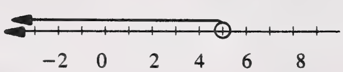
Solution set is $\{x \mid x > -4, x \in R\}$



(c)
$$\frac{6(x + 1)}{3} > \frac{(x - 1)6}{2}$$
$$\begin{aligned} 2x + 2 &> 3x - 3 \\ -x &> -5 \\ x &< 5 \end{aligned}$$

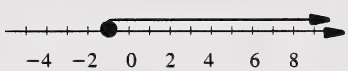
Solution set is $\{x \mid x < 5, x \in R\}$

Graph:



(d)
$$2x + 10 - x \leqq 3x + 12$$
$$\begin{aligned} x + 10 &\leqq 3x + 12 \\ -2x &\leqq 2 \\ x &\geqq -1 \end{aligned}$$

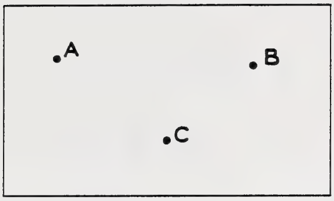
Solution set is $\{x \mid x \geqq -1, x \in R\}$



END OF LESSON 9

LESSON 10

Page 1



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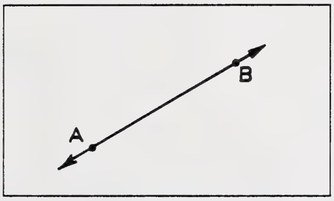

$\overleftrightarrow{\underline{AB}}$, $\overleftrightarrow{\underline{BA}}$

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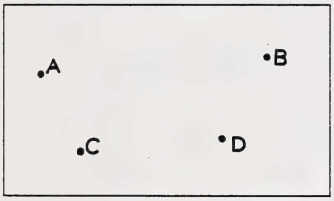

plane $\underline{ABC}$, plane $\underline{BDC}$

Exercise – Points, Lines, Planes

1. (a) line or $\overleftrightarrow{XY}$ (b) point or Point R
   (c) plane or Plane MRS

2. (a) ABC
   (b) Yes
   (c) $\overleftrightarrow{AC}$, $\overleftrightarrow{BC}$
   (d) $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$
   (e) $\overleftrightarrow{AC}$, $\overleftrightarrow{BC}$

3. $\overleftrightarrow{RP}$, $\overleftrightarrow{PR}$, $\overleftrightarrow{PQ}$, $\overleftrightarrow{QP}$

4. (a) Plane BEC
   (b) E
   (c) B
   (e) yes
   (f) D or F or G

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Is point R a subset of $\overleftrightarrow{PR}$? <u>Yes</u>

Is point S a subset of $\overleftrightarrow{PR}$? <u>No</u>

Name this segment. $\underline{\overline{MN}}$

Name its endpoints. <u>M</u>, <u>N</u>

d(M, N) = 4.4 cm

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d(P, Q) = <u>2.5 cm</u>, d(Q, R) = <u>2.5 cm</u>, d(P, R) = <u>5cm</u>

It would also be named $\overrightarrow{RT}$ or $\overrightarrow{RU}$

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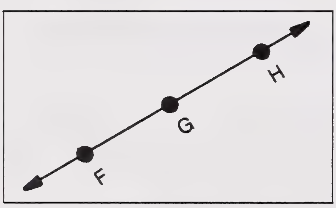


2. $\overleftrightarrow{FG}$

3. $\overline{GH}$, $\overline{FH}$

4. $\overrightarrow{GH}$, $\overrightarrow{HF}$, or $\overrightarrow{HG}$, $\overrightarrow{GF}$

Exercise – Subsets of a Line

1. (a) set, points
   (b) subset, subsets
   (c) point
   (d) length, thickness
   (e) segment
   (f) ray
   (g) points, arrow
   (h) length
   (i) d(R, S)
   (j) endpoint
   (k) collinear

2. (a) Name of line $\overleftrightarrow{XY}$ (b) Name of line $\overleftrightarrow{MN}$

Name of segment $\overline{XY}$ Name of segment $\overline{MN}$

Name of rays $\overrightarrow{YX}$, $\overrightarrow{XY}$ Name of rays $\overrightarrow{MN}$, $\overrightarrow{NM}$

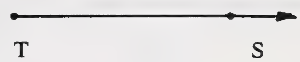
3. (a) $\overrightarrow{AC}$, $\overrightarrow{AD}$, $\overrightarrow{AB}$

(b) $\overline{AB}$, $\overline{AD}$, $\overline{AC}$

(c) no

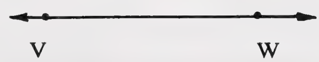
4. (b)

(c)



(d)

(e)



(f)

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5. (a) A, B, C, D, E

(b) $\overleftrightarrow{AC}$, $\overleftrightarrow{BD}$

(c) $\overline{AC}$, $\overline{AE}$, $\overline{EC}$

(d) $\overrightarrow{EA}$, $\overrightarrow{AC}$, $\overrightarrow{EC}$, $\overrightarrow{CA}$ or $\overline{CE}$, $\overline{AE}$ (any four)

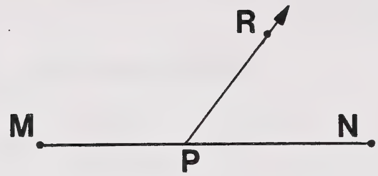
(e) $\overrightarrow{EB}$, $\overrightarrow{EC}$, $\overrightarrow{EA}$, $\overrightarrow{ED}$

(f) $\overline{BE}$, $\overline{BD}$, $\overline{BA}$, $\overline{BC}$ (any two)

(g) E

(h) midpoint

6.



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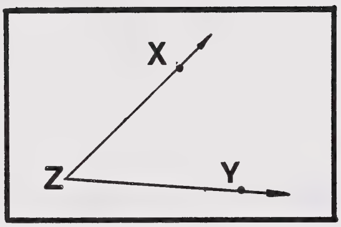

What is the vertex of this angle? <u>**Z**</u>

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Name the two rays that make up this angle. <u>**$\overrightarrow{SR}$**</u>, <u>**$\overrightarrow{ST}$**</u>

What is the common endpoint of these rays? **S**
This common endpoint is called the <u>**vertex**</u> of the angle.

The three angles shown in this diagram are ∠AOC, ∠<u>**AOB**</u> and ∠<u>**BOC**</u>.

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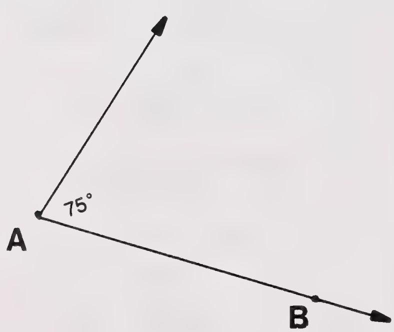

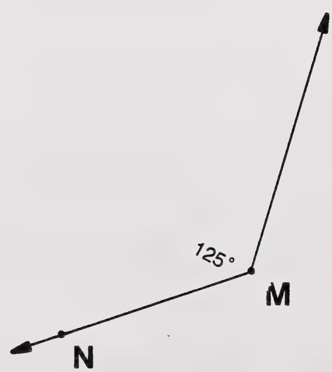

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Exercise – Angles

1. (a) (ii) $\overrightarrow{AB}$

(iii) A

(iv) 70°

(v) acute

(b) (i) ∠PRT

(ii) $\overrightarrow{RT}$, $\overrightarrow{RP}$

(iii) R

(iv) 135°

(v) obtuse

(c) (i) ∠YZV

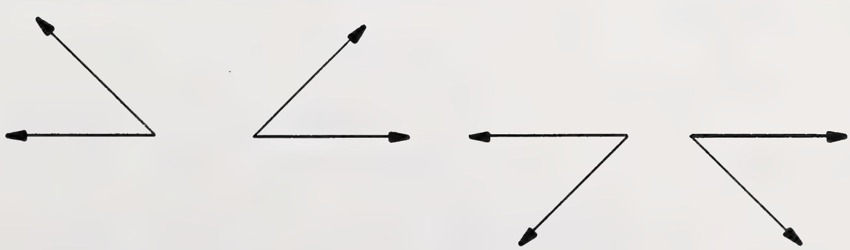
(ii) $\overrightarrow{ZY}$, $\overrightarrow{ZV}$

(iii) Z

(iv) 90°

(v) right

2. Position 1 Position 2 Position 3 Position 4



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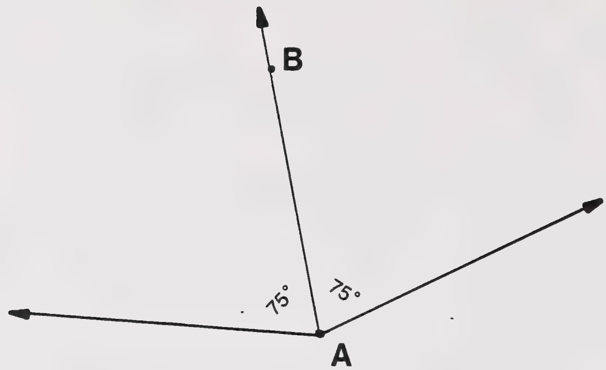
3.



4. (a) $\overleftrightarrow{AC}$, $\overleftrightarrow{MT}$

   (b) $\overline{RM}$

   (c) R

   (d) $\angle$ MRC

   (e) acute

   (f) obtuse

   (g) $\overrightarrow{RC}$, $\overrightarrow{RT}$

   (h) $\overrightarrow{RT}$

   (i) R

   (j) $\overrightarrow{RT}$

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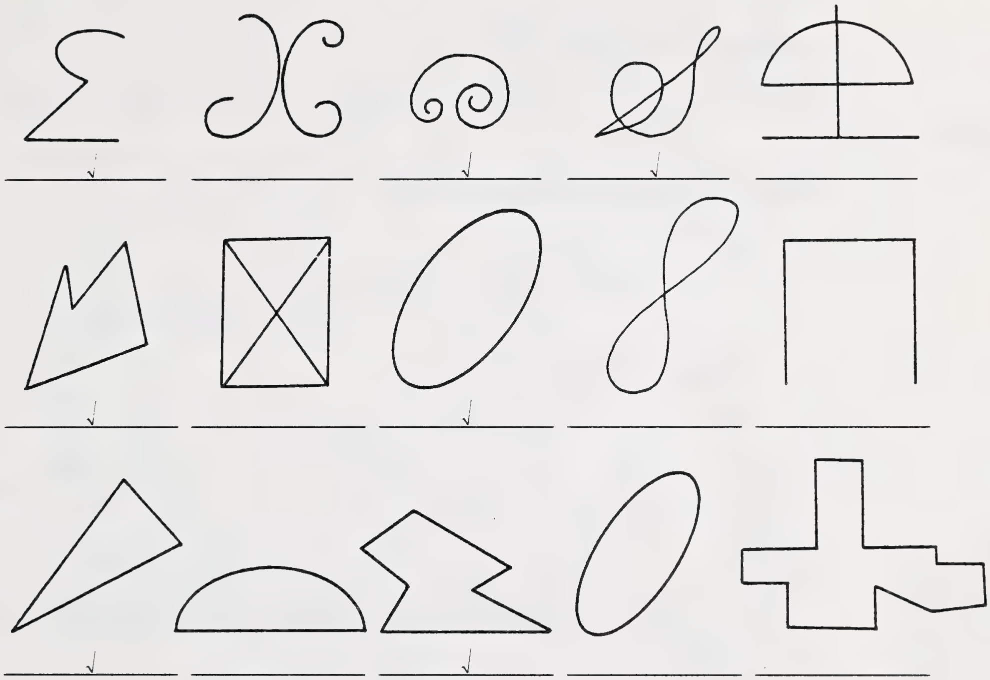

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Its five sides are $\overline{AB}$, **$\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EA}$**

Its five vertices are A, **B, C, D, E.**

Name this polygon. **PQRS.**

The names of these diagonals are $\overline{PR}$ and **$\overline{QS}$.**

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tricycle, tripod (or other examples)

quadruplets, quadrant (or other examples)

A regular hexagon has how many equal sides and angles? **6**
A regular decagon has how many equal sides and angles? **10**
A regular **pentagon** has five equal sides and angles.
A regular **heptagon** has seven equal sides and angles.

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5 cm, 90°

quarilatural, 3.8 cm, 68°, 112°

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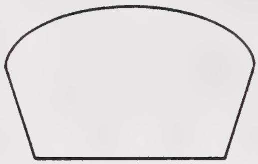
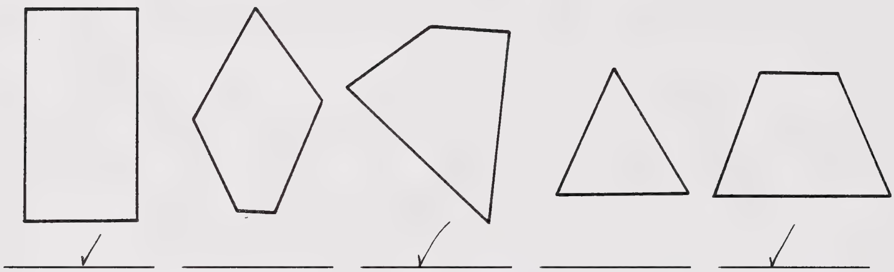
Exercise – Special Kinds of Curves

1.

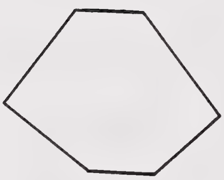


(or any other similar figure)

2.

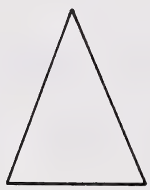


3.



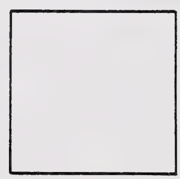
irregular

hexagon



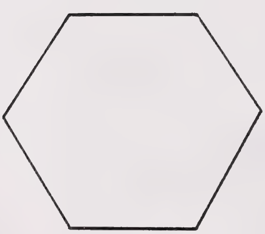
irregular

triangle



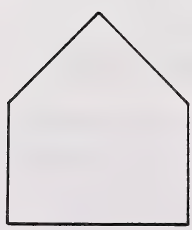
regular

quardrilateral



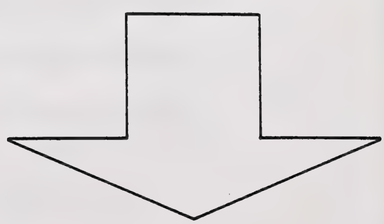
regular

hexagon



irregular

pentagon



irregular

heptagon

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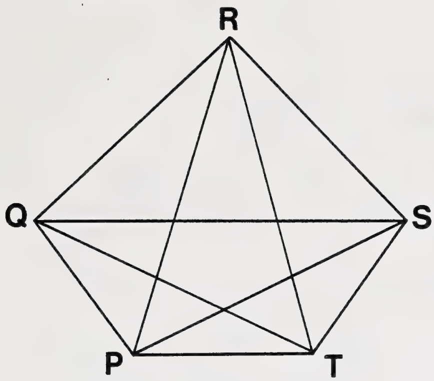

Sides: $\overline{RS}$, $\overline{ST}$, $\overline{TP}$, $\overline{PQ}$, $\overline{QR}$

Diagonals: $\overline{RT}$, $\overline{RP}$, $\overline{QS}$, $\overline{QT}$, $\overline{PS}$

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(a) $\triangle$MTR, $\triangle$MRT, $\triangle$RMT, $\triangle$RTM, $\triangle$TRM, $\triangle$TMR

(b) R, T

(c) $\overline{MT}$ 2.5cm
$\overline{RT}$ 3.9cm
$\overline{RM}$ 4.6 cm

(d) $\angle$MRT 34°

$\angle$RMT 58°

What is the sum of these three measures? **180**

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1. 5.1 cm, 3.9 cm, 3.2 cm

2. 3.8 cm, 2.6 cm

3. 3.3 cm

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1. $\angle$XYZ, 35°, 55°

   Name the hypotenuse of $\triangle$XYZ. $\overline{XZ}$

   The legs of $\triangle$XYZ are $\overline{XY}$ and $\overline{YZ}$.

2. 40°, 60°, and 80°

3. Which angle is obtuse? $\angle$QPR
   What is the measure of this obtuse angle? 120°
   The other two angles measure 40° and 20°

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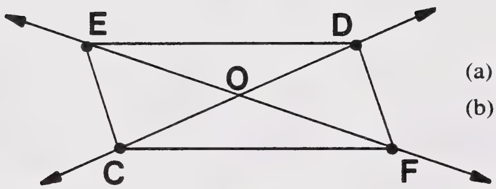
1. (a) (ii) $\overline{GH}$, $\overline{HI}$, $\overline{IG}$
   (iii) G, H, I
   (iv) $\angle G$, $\angle H$, $\angle I$

   (b) (i) $\triangle SRT$
   (ii) $\overline{SR}$, $\overline{RT}$, $\overline{TS}$
   (iii) S, R, T
   (iv) $\angle S$, $\angle R$, $\angle T$

2. $\triangle ABC$ $\quad$ $\triangle QSR$, $\triangle QPT$, $\triangle QTR$

   $\triangle ABR$ $\quad$ $\triangle SPT$, $\triangle STR$, $\triangle RSP$

   $\triangle BRC$

3. 

   (a) $\overrightarrow{OE}$, $\overrightarrow{OD}$, $\overrightarrow{OF}$
   (b) $\triangle CED$, $\triangle EDF$, $\triangle CEF$, $\triangle EOD$, $\triangle DOF$, $\triangle COF$, $\triangle EOC$

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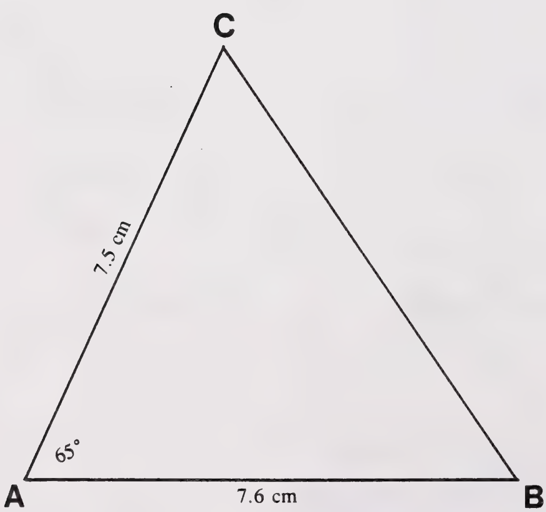

5. (a)

Triangle	Measurement of Side			Classification
	1	2	3	
△ ABC	3.8 cm	3.8 cm	3.8 cm	equilateral
△ DEF	2.6 cm	4.4 cm	5.1 cm	scalene
△ GHI	3.2 cm	3.2 cm	5.7 cm	isosceles

(b)

Triangle	Measurement of			Classification
	∠1	∠2	∠3	
△ ABC	60°	60°	60°	acute
△ DEF	31°	59°	90°	right
△ GHI	29°	29°	122°	obtuse

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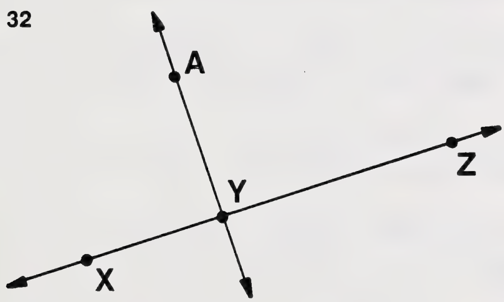

$\overleftrightarrow{XZ} \perp \overleftrightarrow{AY}$

∠AYZ, ∠AYX

Use the symbols to write this. $\overline{AD} \perp \overline{BC}$

Name two right angles that are formed. **∠ADB, ∠ADC**

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Name the two lines that are parallel. $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$

Write this fact symbolically. $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

Name the two lines that are perpendicular. $\overleftrightarrow{BD}$, $\overleftrightarrow{AB}$

Write this fact symbollically. $\overleftrightarrow{BD} \perp \overleftrightarrow{AB}$

Name two segments that are parallel in this figure. $\overline{PS}$, $\overline{QR}$

Write this fact symbolically. $\overline{PS} \parallel \overline{QR}$

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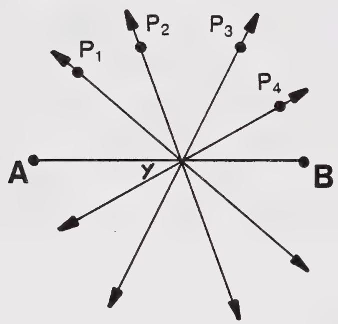
(d) (Y,z) = **2.6 cm**



What is the midpoint of $\overline{AB}$? **Y**

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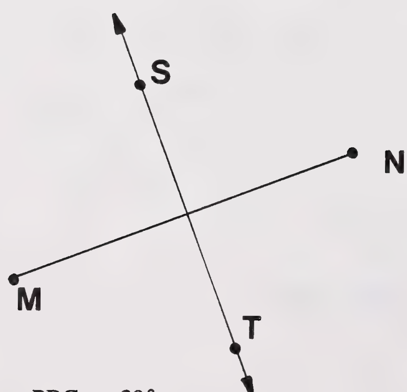

m $\angle$PBC = **30°**
m $\angle$PQR = **120°**
m $\angle$PQY = m $\angle$**RQY** = **60°**

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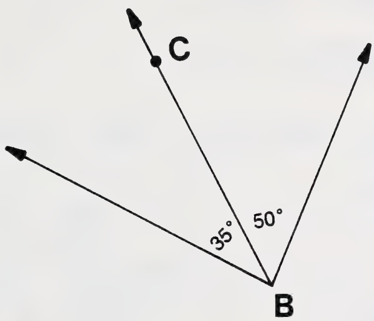
Name the common vertex. **Q**

Name the common arm. $\overrightarrow{\mathbf{QS}}$



m ∠ABC = 30°, m ∠DEF = 60°
m ∠ABC + m ∠DEF = 30° + 60° = 90°

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∠XOY and ∠YOZ are adjacent angles because they have the common vertex **0** and the common arm $\overrightarrow{\mathbf{OY}}$. They are complementary because the sum of their measures is **90°**

m ∠XOY + m ∠YOZ = **35°** + **55°** = **90°**
What kind of angle is ∠XOZ? **right**

m ∠JKL = **120°**, m ∠MNO = **60°**
m ∠JKL + m ∠MNO = **120° + 60° = 180°**

∠ABC and ∠**ABD** are adjacent angles because they have the common vertex **B** and the common arm $\overrightarrow{\mathbf{BA}}$. They are supplementary because the sum of their measures is **180°**.

m ∠ABC + m ∠ABD = **82° + 98° = 180°**

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m ∠ACB = **50°**
m ∠BAC = **70°**

∠ABD and ∠**CBE** are also vertical angles. They have the same vertex **B.** $\overrightarrow{\mathrm{BA}}$ and $\overrightarrow{\mathbf{BE}}$ are opposite rays, white $\overrightarrow{\mathrm{BD}}$ and $\overrightarrow{\mathbf{BC}}$ are opposite rays.

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m $\angle$CBE + m $\angle$EBD = **180°**
m $\angle$EBD + m $\angle$ABD = **180°**
m $\angle$CBE + m $\angle$EBD = m $\angle$EBD + **m $\angle$ABD**

We can subtract **m $\angle$EBD...**
m $\angle$CBE = **m $\angle$ABD**

$\angle$ACE = $\angle$ECB = **$\angle$ACD** = **$\angle$BCD** = **$\angle$BCE**

$\triangle$MNO = $\triangle$MON = **$\triangle$OMN** = **$\triangle$ONM** = **$\triangle$NMO**

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d(A, B) = **4.5 cm**, d(C, D) = **4.5 cm**

m $\angle$ABD = **50°**, m $\angle$DBC = **50°**

d(A, B) = **5.1** cm, d(B, D) = **4.5** cm
d(C, D) = **5.1** cm, d(A, C) = **4.5** cm

Thus $\overline{AB} \cong \overline{CD}$ and $\overline{BD} \cong \overline{CA}$

m $\angle$A = **110°**, m $\angle$B = **70°**, m $\angle$C = **70°**, m $\angle$D = **110°**

$\angle$A $\cong$ $\angle$**D** and $\angle$**B** $\cong$ $\angle$**C**

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Sides	Angles
$\overline{AB} \cong \overline{TS}$	$\angle C \cong \angle R$
$\overline{BC} \cong \overline{SR}$	$\angle B \cong \angle S$
$\overline{CD} \cong \overline{RQ}$	$\angle A \cong \angle T$

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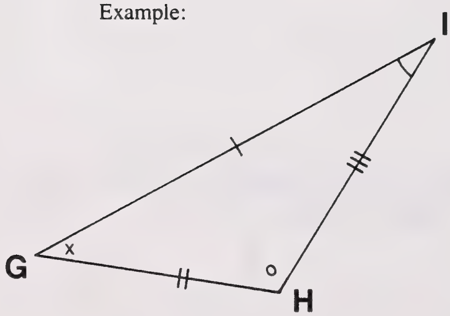
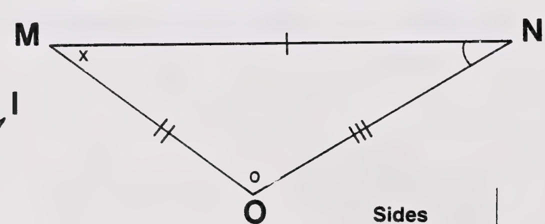
Example:





Sides	Angles
$\overline{GH} \cong \overline{MO}$	$\angle G \cong \angle M$
$\overline{HI} \cong \overline{ON}$	$\angle I \cong \angle N$
$\overline{GI} \cong \overline{MN}$	$\angle H \cong \angle O$

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Sides	Angles
$\overline{XY} \cong \overline{LK}$	$\angle X \cong \angle L$
$\overline{XZ} \cong \overline{LM}$	$\angle Y \cong \angle K$
$\overline{YZ} \cong \overline{KM}$	$\angle Z \cong \angle M$

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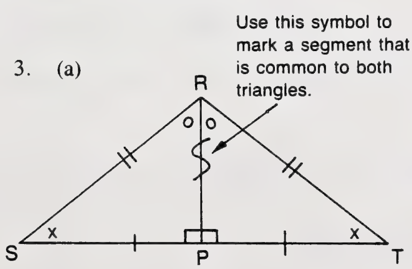
Exercise – Other Geometric Terms

1. (a) right
   (b) acute
   (c) complementary
   (d) perpendicular
   (e) protractor
   (f) measure, angle
   (g) sides, angles
   (h) vertices
   (i) bisector, perpendicular bisects
   (j) adjacent
   (k) equilateral
   (l) right
   (m) hypotenuse
   (n) measure
   (o) linear

2. Let measure of first angle be $x°$.
   Then measure of second angle is $(x + 10)°$

$$
\begin{aligned}
x + x + 10 &= 180 \\
2x &= 170 \\
x &= 85 \\
\therefore\ x &= 85° \\
x + 10 &= 95°
\end{aligned}
$$

3. (a)



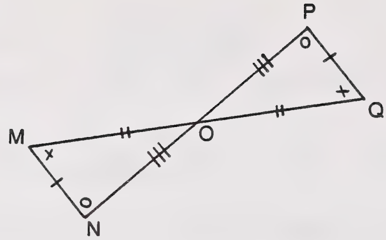
$\triangle \underline{RSP} \cong \triangle \underline{RTP}$

(i) $\overline{RS} \cong \underline{\overline{RT}}$ (iv) $\angle SRP \cong \angle \underline{TRP}$

(ii) $\overline{SP} \cong \underline{\overline{TP}}$ (v) $\angle RPS \cong \angle \underline{RPT}$

(iii) $\overline{RP} \cong \underline{\overline{RP}}$ (vi) $\angle RSP \cong \angle \underline{RTP}$

(b)



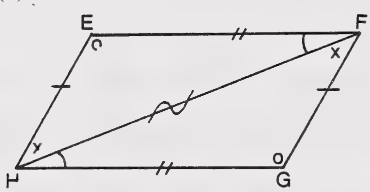
$\triangle OMN \cong \triangle OQP$

(i) $\overline{OM} \cong \overline{OQ}$ (iv) $\angle M \cong \angle Q$

(ii) $\overline{MN} \cong \overline{QP}$ (v) $\angle N \cong \angle P$

(iii) $\overline{ON} \cong \overline{OP}$ (vi) $\angle MON \cong \angle QOP$

(c)



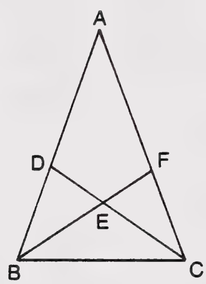
$\triangle EHF \cong \triangle GFH$

(i) $\overline{EH} \cong \overline{GF}$ (iv) $\angle E \cong \angle G$

(ii) $\overline{HF} \cong \overline{FH}$ (v) $\angle EHF \cong \angle GFH$

(iii) $\overline{EF} \cong \overline{GH}$ (vi) $\angle EFH \cong \angle GHF$

(d)



$\triangle DEB \cong \triangle FEC$

(i) $\overline{DE} \cong \overline{FE}$ (iv) $\angle D \cong \angle F$

(ii) $\overline{EB} \cong \overline{EC}$ (v) $\angle DEB \cong \angle FEC$

(iii) $\overline{DB} \cong \overline{FC}$ (vi) $\angle B \cong \angle C$

or $\triangle ADC \cong \triangle AFB$

(i) $\overline{AD} \cong \overline{AF}$ (iv) $\angle A \cong \angle A$

(ii) $\overline{DC} \cong \overline{FB}$ (v) $\angle D \cong \angle F$

(iii) $\overline{AC} \cong \overline{AB}$ (vi) $\angle C \cong \angle B$

or $\triangle DBC \cong \triangle FCB$

(i) $\overline{DB} \cong \overline{FC}$ (iv) $\angle D \cong \angle F$

(ii) $\overline{BC} \cong \overline{CB}$ (v) $\angle B \cong \angle C$

(iii) $\overline{DC} \cong \overline{FB}$ (vi) $\angle C \cong \angle B$

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4. (a) ray XY
   (b) segment MN
   (c) angle ABC
   (d) triangle PQR
   (e) line BC
   (f) measure of angle C
   (g) is perpendicular to
   (h) is parallel to
   (i) distance M to N
   (j) is congruent to

5. (a) false
   (b) true
   (c) true
   (d) false
   (e) true
   (f) false
   (g) true
   (h) false
   (i) true
   (j) false

6. (a) $\angle PQY$, $\angle RQY$
   (b) $\angle PYQ$, $\angle RYQ$
   (c) $\angle PYQ$, $\angle SYR$
   (d) $\overline{PQ}$, $\overline{SR}$
   (e) $\overline{PQ}$, $\overline{PS}$
   (f) $\overline{PQ}$, $\overline{SR}$
   (g) $\angle PSR$
   (h) $\overline{RP}$

   Other answers are exceptable, be sure they are comparable.

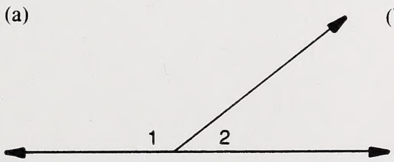
7. (a) $\triangle ABC \cong \triangle JKL$
   (b) $\overline{XY} \parallel \overline{MN}$
   (c) $m \angle PQR = 30°$
   (d) $d(G, H) = 3cm$
   (e) $MN \perp MW$

8. (a)
$$m \angle BDF = 180 - 123 = 57°$$
$$m \angle DBF = 180 - 57 - 51 = 72°$$
$$m \angle ABC = m \angle DBF$$
$$\therefore m \angle ABC = 72°$$

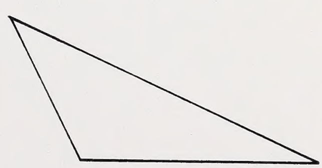
   (b)
$$m \angle FBG = 64°$$
$$m \angle BEG = 72°$$
$$m \angle BGE = 180 - 64 - 72$$
$$= 44°$$

9. (a) The have a common vertex B, and a common ray $\overrightarrow{BF}$.
   (b) The two angles have a degree total of 90°
   (c) $90 - y$

10. (a)



(b)



END OF LESSON 10